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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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COLOR INDICES AND INTEGRATED MAGNITUDES OF FIFTEEN BRIGHT GLOBULAR CLUSTERS

By A. N. VYSSOTSKY and EMMA T. R. WILLIAMS

ABSTRACT

Extra-focal photographs of fifteen globular clusters were obtained in blue and in yellow light with two short-focus cameras. It is found that the lower the galactic latitude of a cluster, the redder is its color. The increase in color corresponds to a differential absorption of $0^{m}11 \pm .02$ encountered by light passing perpendicularly through the Milky Way. The derived integrated apparent magnitudes of the clusters lead to integrated absolute magnitudes which, on the average, are 2 mag. brighter than the integrated absolute magnitudes of Hubble's objects connected with M 31 which he has provisionally identified as globular clusters.

A study of the colors of the globular clusters may contribute to a better knowledge of the absorbing medium in the galactic system. The integrated magnitudes have particular significance in comparing the globular clusters connected with our galaxy with those objects provisionally identified as globular clusters connected with other galaxies. The present work was well advanced when we learned that Stebbins had attacked the same problem by a different method;[1] consequently we have not extended our program to fainter clusters.

OBSERVATIONAL METHODS

The chief difficulties in obtaining color indices of globular clusters by photography are (*a*) the non-stellar appearance of the clusters and (*b*) the lack of carefully determined colors of the field stars.

To overcome the first difficulty, two very short-focus cameras were used extra-focally. A Schneider Xenon lens of focal ratio 1 : 1.8 and scale of $31'.1$ to the millimeter was used with a yellow filter

[1] *Proceedings of the National Academy of Sciences*, **19**, 222, 1933.

(Wratten No. 12) and Iso Presto plates to obtain photovisual comparisons of the cluster with neighboring stars in the same field. A Zeiss Tessar lens of focal ratio 1:4.8 and scale of 29ʹ.4 to the millimeter was used with Eastman-40 plates to obtain similar photographic comparisons. The two cameras were mounted together on the 6ʺ visual telescope, and exposed simultaneously, with exposures of twenty minutes, for the brightest clusters, up to eighty minutes for the fainter ones. The extra-focal images of the stars ranged from 0.18 mm in diameter on some plates to 0.35 mm on others, depending on the character of the particular cluster in the field. As Plate XI shows, the extra-focal images of the clusters are closely comparable to the extra-focal star images, although they are distinguishable. Three pairs of plates were usually obtained for each cluster. The surface densities of the central area of the images were measured on the Schilt microphotometer, the diameter of this central area being about 0.13 mm.

To determine the colors of the field stars, three photovisual and three photographic extra-focal comparisons with the North Pole were made by means of a 6ʺ Voigtländer lens, the plates being measured on the Schilt microphotometer. The photovisual magnitudes thus obtained were combined with equal weight: (*a*) with the magnitudes of the Harvard Revised Photometry reduced to the International Photovisual System[2] and (*b*) with the magnitudes of the Potsdam Photometry reduced to the International Photovisual System[3] whenever Potsdam magnitudes were available. The photographic magnitudes were combined in the few cases which were possible, with the photographic magnitudes of the *Göttingen Aktinometrie* reduced to the International Photographic System by adding a correction of $0^m.05$.[4] However, since the determination of these magnitudes in general was weak, each was combined with equal weight with a "derived" photographic magnitude obtained by adding the color indices used in the *Henry Draper Catalogue* to the adopted photovisual magnitudes.[5] The resulting colors of the comparison

[2] *Harvard Annals*, **89**, 1, 1931; *Astrophysical Journal*, **61**, 284, 1925.

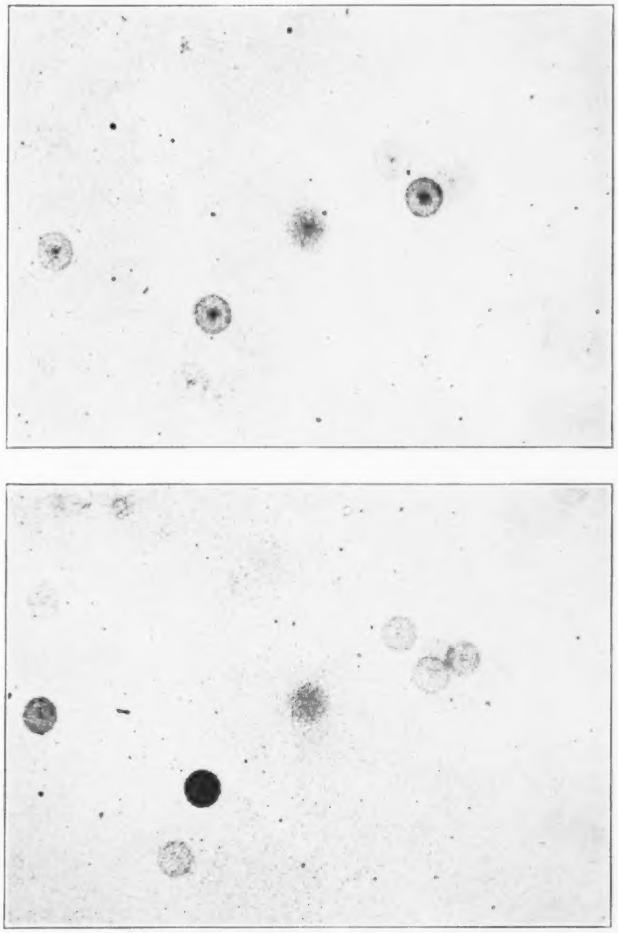
[3] *Astrophysical Journal*, **74**, 131, 1931.

[4] *Transactions of the International Astronomical Union*, **1**, 75, 1922.

[5] In general, the comparison stars were not faint enough to be much affected by space reddening.

PLATE XI

N



EXTRA-FOCAL PHOTOGRAPHS OF MESSIER 22

Enlarged fourteen times, photovisual above, photographic below. M 22 has the largest angular diameter of all the clusters on the program so that the difference between the cluster image and the star images is most pronounced in this case.

stars were grouped according to spectral class and were found to be close to the *HDC* colors, as is shown in the accompanying tabulation.

Spectral class	B8–A3	A5–F2	F5–G0	G5–K2
Number of stars	37	13	11	31
HDC C.I. *minus* McC. C.I.	+.03	+.01	+.04	−.12

By grouping the comparison stars according to field, it is found that the derived colors of the clusters average $0^{m}.03$ redder than the colors expressed in the *HDC* system.

SURFACE DENSITIES

Table I summarizes the observations. The data are arranged in order of galactic latitude. The fourth column gives the average diameter of the star images on the Zeiss and Schneider plates. In the case of M 5 and M 15 the diameters were made rather small in order to avoid the images of the field stars over lapping the cluster image. The fifth column contains the diameter of the cluster, as given by Shapley and Sawyer,[6] reduced to millimeters on the scale of the Zeiss and Schneider plates. The sixth column contains the concentration class as given by Shapley and Sawyer.[7] The seventh and eighth columns give the surface densities expressed in magnitudes; thus, for M 53 the extra-focal surface density of the cluster in photovisual light is equal to that of a star of magnitude 7.94. In this particular case, since the diameter of the cluster is considerably smaller than the diameters of the extra-focal star images, these measures of surface density are approximately equal to the integrated *pv* and *pg* magnitudes of the cluster. On the contrary, with M 5 and M 15 they are fainter than the magnitudes of the clusters. The internal accuracy of these surface densities is high, the individual residuals from ninety-one plates averaging $0^{m}.05$, so that the formal probable error of each mean is $0^{m}.024$.

COLOR INDICES

The ninth column contains the color index derived from the seventh and eighth columns. These color indices are affected by small systematic errors arising from (*a*) the slight difference in scale of the Zeiss and Schneider plates; (*b*) any difference in diameters of

[6] *Harvard Bulletin*, **852**, 1927. [7] *Ibid.*, **849**, 1927.

the extra-focal images between the Zeiss and Schneider plates of the same region (disregarding sign, these averaged about 0.015 mm); (*c*) any difference in distribution of light within the Zeiss extra-focal pattern as compared with the Schneider extra-focal pattern; and (*d*) any difference in the distance corrections of the two lenses which had not been properly allowed for. To determine the size of these errors, two photographic plates each of M 3 and M 13 were obtained with the Schneider lens. It was thought that M 3 would show

TABLE I

Messier Number (1)	N.G.C. (2)	Gal. Lat. (3)	Diam. of Star Images (4)	Diam. of Cluster (5)	Concentration Class (6)	Surface Density Pv (7)	Surface Density Pg (8)	Color Index (9)	Harvard Spect. Class (10)	Integrated Pg Mag. McC. (11)	Integrated Pg Mag. Harvard (12)	Integrated Pg Mag. Vor.-V. (13)
			mm	mm								
53	5024	+79°	0.19	0.11	V	7^m94	8^m21	0.27	*	8.1	6.9	.....
3	5272	+78	.20	.33	VI	6.59	7.15	0.56	G	6.8	4.5	.....
5	5904	+46	.18	.42	V	6.49	7.04	0.55	G:	6.6	3.6	.....
13	6205	+40	.25	.33	V	6.13	6.81	0.68	G0	6.7	4.0	.....
2	7089	−36	.18	.27	II	6.53	7.32	0.79	F5	7.1	5.0	.....
92	6341	+35	.23	.28	IV	6.59	7.15	0.56	G5:	7.0	5.1	.....
15	7078	−28	.16	.25	IV	6.56:	7.32:	0.76:	F	7.0	5.2	.....
55	6809	−25	.26	.33	XI	6.87	7.61	0.74	.....	7.4	4.4	.....
12	6218	+25	.29	.31	IX	7.18	7.98	0.80	.....	7.9	6.0	8.22
10	6254	+22	.29	.27	VII	6.90	7.78	0.88	†	7.6	5.9	8.05
4	6121	+15	.29	.47	IX	6.43	7.47	1.04	F	7.2	5.2	7.45
22	6656	− 9	.32	.58	VII	5.52	6.24	0.72	.....	5.9	3.6	6.5
19	6273	+ 9	.20	.14	VIII	7.12	7.90	0.78	G5:	7.8	6.8	7.67
62	6266	+ 7	.20	.14	IV	6.88	8.00	1.12	K0	7.9	7.0	8.02
28	6626	− 7	0.20	0.16	IV	7.14	8.08	0.94	G5	8.0	6.8	8.06

* Dark lines $H\delta$, H, and K faintly seen.
† Dark lines in the violet appear to be H, K, and $H\zeta$.

the maximum error and M 13 the average error. The photographic surface densities from the Schneider plates are 7.18 for M 3 and 6.84 for M 13 as compared with 7.15 and 6.81 obtained from the Zeiss plates, hence we have neglected systematic errors of this type. The probable error in the color indices computed from the internal agreement of the observations is 0^m04.

DIFFERENTIAL ABSORPTION OF LIGHT IN SPACE

A glance down the ninth column immediately indicates that the colors of the clusters grow redder with decreasing galactic latitude. It might be possible to attribute this increase in color to progressive

change in spectral class.[8] However, there appears to be little correlation between the colors in column 9 and the spectral class assigned by Miss Cannon[9] which is given in the tenth column. Furthermore, Fath's investigations of globular cluster spectra[10] and recent unpublished work at Mount Wilson, according to information obtained from Director Adams, both point to the conclusion that the predominant spectral class is F.

If, on the other hand, we assume that the increase in color is due to a differential absorption in a layer of limited thickness symmetrical with the galactic plane, we can write the equation:

$$\text{Observed } C.I. = \text{Intrinsic } C.I. \text{ of cluster} + D \csc |b|$$

or

$$C_o = C_i + D \csc |b| ,$$

where b is galactic latitude and $2D$ is the differential absorption in magnitudes, encountered by light passing perpendicularly through the Milky Way, i.e., $2D$ is the difference between the optical thickness of the galactic absorbing layer in blue light and that in yellow light. Solving the fifteen observation equations by least squares, we get

$$\begin{aligned} C_o &= 0.56 + 0.055 \csc |b| . \\ &\pm .05 \pm .011 \end{aligned}$$

According to this solution, the intrinsic color index of the globular clusters is $+0.56 \pm .05$, and the total differential absorption in the galactic absorbing layer is $0^{m}.11 \pm .02$, which is only about half as much as the differential absorption of light passing perpendicularly through the earth's atmosphere. It is interesting to note that van de Kamp's derivation of $2D$ from less reliable colors of globular clusters is $0^{m}.08 \pm .06$.[11]

If Rayleigh scattering were the only type of obscuration in space, then it would be possible to compute the optical thickness of the absorbing layer in photographic and in photovisual light from the value of the differential absorption found above. Thus, the optical

[8] Cf. Shapley, *Proceedings of the National Academy of Sciences*, **19**, 30, 1933.

[9] *Harvard Bulletin*, **868**, 1929.

[10] *Astrophysical Journal*, **33**, 58, 1911; **37**, 198, 1913.

[11] *Astronomical Journal*, **40**, 156, 1930.

thickness of the absorbing layer in photographic light would be $0^m.17 \pm .03$, and that in photovisual light would be $0^m.06 \pm .01$. Since the value of the optical thickness of the absorbing layer in photographic light, as actually derived from the distribution of extragalactic nebulae, is $0^m.8 \pm .2$,[12] it is legitimate to infer that Rayleigh scattering constitutes only a minor part of the total obscuration, most of which must be done by particles larger in diameter than the wave-length of blue light. It is, nevertheless, interesting to note that in a column of 1 sq. cm cross-section passing perpendicularly through the Milky Way, the number of small solid particles sufficient to produce the Rayleigh scattering, computed in a manner similar to that used by Gleissberg[13] is found to be in the neighborhood of 10^{18} or 10^{19}.

COMPARISON WITH STEBBINS' RESULTS

The value found above for the total differential absorption in the galactic absorbing layer, $0^m.11 \pm .02$, differs considerably from the corresponding figure found by Stebbins, namely, $0^m.18 \pm .01$. It is a simple matter to reconcile the two values, however. The difference between them is due largely to the fact that they are based on different systems of color indices. The difference in color index between A0 and K0 for the comparison stars used in the present paper is $1^m.15$. The corresponding difference in the system used by Stebbins is $1^m.48$. Hence to make the differential absorption values comparable, it is necessary to introduce the factor 1.15/1.48. Multiplying Stebbins' value by this, we obtain $0^m.14 \pm .01$, which agrees excellently with the value $0^m.11 \pm .02$ found herein. It may be suspected that this is a not quite legitimate procedure since the value $1^m.48$ is the difference in color index between A0 stars and K0 *giant* stars as obtained by Seares,[14] whereas the value $1^m.15$ is the difference in color index between A0 stars and a mixture of K0 giants and dwarfs. However, from the spectroscopic researches of Lindblad, Petersson, and Schalén[15] in various galactic latitudes from 50° to 0°, it is abundantly

[12] *Ibid.*, **42**, 97, 1932. [13] *Astronomische Nachtrichten*, **246**, 329, 1932.

[14] *Astrophysical Journal*, **55**, 198, 1922.

[15] *Nova acta regiae scient. Upsaliensis*, Ser. IV, **6**, No. 5, 1925; *Meddelanden Från. Astronomiska Observatorium, Upsala*, No. 11, 1926; *ibid.*, No. 29, 1927; *ibid.*, No. 55, 1931.

evident that among stars brighter than the tenth apparent magnitude the ratio of giants to dwarfs in classes G8–K2 is practically always greater than 6 to 1. Hence *the effect of the slight admixture of dwarfs is negligible.*

As to which is the more desirable system of color indices, it may be well to point out that Seares, in publishing the colors used by Stebbins, referred to them as "provisional," and that they were derived from the color indices of about one hundred and fifty stars. From more recent work by Seares, Sitterly, and Joyner,[16] involving more than three hundred stars, the difference in color index between A0 and K0 stars is $1^{m}.03$. Furthermore, from the work of Miss Payne[17] on more than eight hundred stars in the Harvard standard regions, the difference in color index between A0 and K0 is about $1^{m}.04$. Again, the difference obtained by Ross and Zug[18] from more than eight hundred stars is $1^{m}.10$. There are, of course, many more investigations which might be mentioned, but these are probably representative for stars brighter than the tenth magnitude. It seems justifiable, then, to convert Stebbins' value for differential absorption to our scale rather than to convert our value to his scale.

It is interesting to determine the probable errors in the individual color indices of clusters by using the ten clusters which are common to the two lists. It is found that the average difference, disregarding sign, between our color indices and those of Stebbins reduced to the system used in this paper is $0^{m}.13$. If we assume that the two sets are of approximately equal accuracy, this corresponds to a probable error in one determination of color index of $0^{m}.08$.

INTEGRATED MAGNITUDES

The eleventh column of the table gives the integrated photographic magnitude of each cluster derived as follows. From Hogg's measures of the distribution of light within globular clusters[19] was computed the fraction of the total light of the extra-focal image of the cluster which falls within 0.065 mm of the center (since the area measured by the microphotometer was 0.065 mm in radius). This

[16] *Astrophysical Journal*, **72**, 311, 1930.

[17] *Harvard Bulletin*, **881**, 18, 1931.

[18] *Astronomische Nachrichten*, **239**, 289, 1930.

[19] *Astronomical Journal*, **42**, 77, 1932.

was compared with the corresponding fraction for the star images, and corrections were derived to convert the surface densities into integrated magnitudes. The larger the ratio between the extra-focal image and the diameter of the cluster, the smaller is the derived correction. These values were checked by measuring the plates in the microphotometer with a large diaphragm, so that an area 0.40 mm in diameter was included. Incidentally, this involved a slight rearrangement of the microphotometer, since the area of the thermopile is not large enough to receive light from so large an area. These measures are reliable enough to serve as an independent check on the "derived" values, in spite of the fact that they show a marked dependence on the density of the images in the sense which is to be expected. Thus, since the intensity in the center of the extra-focal image of the cluster is very much greater than that in the outer fringe, the range in intensity is greater than the straight-line portion of the characteristic curve of the photographic plate, and the result of this is that the denser the image, the brighter the magnitude measured for the cluster.

It is of interest to compare these values with those estimated by Shapley and Sawyer,[20] which are given in the twelfth column, and with those of Vorontsov-Velyaminov[21] in the thirteenth column. The magnitudes derived here agree fairly well with those of Vorontsov-Velyaminov, being on the average $0^{m}.2$ brighter. The modifications which they indicate in the Shapley-Sawyer scale are given herewith.

Shapley-Sawyer integrated magnitude.....	4	5	6	7
McCormick integrated magnitude........	6.6	7.1	7.6	8.1
Hubble integrated magnitude............	7.8	8.4	8.8	9.2

For comparison, Hubble's revision[22] of the Shapley-Sawyer magnitudes is given in the third line, from which it is seen that the McCormick scale parallels that of Hubble but differs from it systematically by 1 mag. It is difficult to see how the McCormick integrated magnitudes can be systematically too bright by as much as half a magnitude. The McCormick apparent integrated magnitudes lead to absolute integrated magnitudes ranging from -7.1 to -8.6, and aver-

[20] *Harvard Bulletin*, **848**, 1927.

[21] *Astronomische Nachrichten*, **236**, 1, 1929.

[22] *Astrophysical Journal*, **76**, 44, 1932.

aging -8.0. These are a magnitude brighter than those derived by Hubble for comparison with the magnitudes of the nebulous objects in M 31 which he identifies provisionally as globular clusters, and they increase the discrepancy between the absolute magnitudes of the globular clusters and those of Hubble's objects to something over 2 magnitudes.

We wish to express our thanks to Dr. A. L. Bennett who designed and constructed the camera for the 2-inch Schneider lens.

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UNIVERSITY OF VIRGINIA
February 1933

THE ORBITS OF TWO SPECTROSCOPIC BINARIES[1]

By WILLIAM H. CHRISTIE

ABSTRACT

The elements of the orbits of two single-spectrum spectroscopic binaries are given, and their respective velocity-curves, derived from least-squares solutions, are shown.

Boss 9.—The elements derived from measures of thirty-six single-prism spectrograms, grouped into twelve normal places, are: $P=96.41$ days, $K=23.88$ km/sec., $\gamma=1.11$ km/sec., $\omega=222\overset{\circ}{.}1$, $e=0.124$.

Boss 283.—Measures of twenty-seven single-prism spectra, grouped into eleven normal places, form the basis from which the following elements were derived: $P=9.07504$ days, $K=51.07$ km/sec., $\gamma=10.35$ km/sec., $\omega=122\overset{\circ}{.}1$, $e=0.333$.

BOSS 9=5 CETI

$\alpha(1900)$ $0^h0^m.2$	Spectral type, K2
$\delta(1900)$ $-3°0'.0$	Absolute magnitude, -1.5

The binary character of Boss 9 was announced by Adams[2] in 1914, and this star was selected for investigation by the writer in 1929. No difficulty was encountered in determining the period, although it was not possible to fill in the velocity-curve as well as might be desired. This was due to the fact that only two cycles of variation were observable in a season and that when the star was between the phases of sixty and seventy days either the telescope was unavailable for spectroscopic work or observations were otherwise unobtainable. It is doubtful, however, whether observations obtained at this phase would materially affect the resulting elements.

The period, 96.41 days, may be considered fairly well established owing to the existence of four plates taken in 1913–1914, which extend the interval between the first and last observations to seventy-two complete cycles. The period, therefore, was not included in the least-squares solution for the most probable values of the elements of the orbit, but was adopted as final.

The thirty-six observations, given in Table I, were grouped into twelve normal places, weighted according to the number and quality

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 469.

[2] *Publications of the Astronomical Society of the Pacific,* **26,** 261, 1914.

of the observations forming the group. These normal places are given in Table II.

TABLE I

OBSERVATIONS OF BOSS 9

Plate	Date	G.M.T.	Phase	Velocity
			days	km/sec.
γ 2773*	1913 Oct. 19	20^h04^m	54.00	+10.4
2886*	Nov. 15	18 13	80.92	−15.4
2917*	Dec. 4	15 16	3.39	−11.7
3038*	1914 Jan. 6	14 39	36.36	+16.8
16750	1929 July 22	23 48	24.55	+20.0
16756	July 23	23 13	25.53	+18.2
16794	Aug. 15	23 30	48.54	+20.3
16801	Aug. 16	22 25	49.50	+19.7
16963	Oct. 9	20 35	7.01	− 7.4
16972	Oct. 10	21 44	8.06	− 6.1
17062	Nov. 14	17 38	42.88	+18.5
17071	Nov. 15	18 49	43.93	+23.6
17158	Dec. 22	15 25	80.79	−21.3
17166	Dec. 23	15 33	81.80	−18.5
17587	1930 July 13	23 52	91.33	−25.8
17597	July 15	23 35	93.31	−25.1
17646	Aug. 10	22 26	22.85	+16.4
17704	Aug. 18	23 12	30.89	+18.3
17711	Aug. 19	22 32	31.86	+23.8
17749	Sept. 7	19 54	50.75	+13.7
17753	Sept. 8	20 26	51.77	+10.8
17831	Oct. 30	17 51	7.25	− 4.4
17844	Nov. 2	16 19	10.19	+ 3.0
17910	Nov. 30	16 01	38.18	+22.0
17920	Dec. 1	16 30	39.20	+18.8
18361	1931 July 28	23 49	85.68	−19.3
18476	Oct. 1	17 20	54.00	+15.6
18490	Oct. 3	18 40	56.06	+ 7.4
18496	Oct. 19	17 05	71.99	−13.5
18500	Oct. 21	18 02	74.03	−16.8
18510	Oct. 22	19 28	75.09	−21.4
18550	Nov. 17	17 18	4.59	−14.4
18618	Dec. 29	14 55	46.49	+23.6
18941	1932 July 16	22 59	54.01	+ 9.5
18949	July 17	23 02	55.01	+ 8.1
18986	Aug. 12	21 57	80.96	−22.1
19061	Sept. 8	20 23	11.49	+ 3.9
19179	Oct. 10	18 56	43.43	+22.7
19250	Nov. 8	16 43	72.34	−12.5
19397	1933 Jan. 3	14 15	31.82	+21.0

* Not included in solution.

The approximate elements were obtained by comparing the plotted observations with standard velocity-curves computed for every 10° in the longitude of periastron and for all eccentricities be-

tween 0.0 and 0.7 in steps of 0.1. Five successive ephemerides were then computed, the elements being changed each time in a manner indicated by the residuals, until the sum of the squares of the residuals was satisfactorily small. The elements for the last of these approximations were then made the basis of a least-squares solution

TABLE II

NORMAL PLACES, BOSS 9

Phase	Velocity	Weight	O−C
days	km/sec.		km/sec.
4.60	−14.4	0.2	−3.18
7.44	− 6.0	0.4	+0.22
10.85	+ 3.5	0.3	+3.34
24.32	+18.3	0.4	−0.61
31.53	+21.0	0.7	−1.76
38.65	+20.4	0.4	−1.71
44.19	+22.1	0.7	+2.57
50.15	+16.0	0.8	+1.05
54.78	+10.2	1.0	−0.14
73.37	−16.0	1.0	−2.33
82.32	−20.3	0.7	+2.80
92.32	−25.4	0.4	−2.23

for the most probable values of K, γ, e, ω, and T. The resulting corrections reduced the sum of the squares of the residuals from 42.91 to 27.19, or about 37 per cent.

TABLE III

ORBITAL ELEMENTS, BOSS 9

Preliminary	Final
$P=96.41$ days	96.41 days
$K=24.0$ km/sec.	23.88 ± 0.90 km/sec.
$\gamma=+1.0$ km/sec.	$+1.11$ km/sec.
$e=0.20$	0.124 ± 0.034
$\omega=220°$	$222.^{\circ}1\pm9.^{\circ}2$
$T=$J.D. 2420007.00	J.D. 2420006.84 ± 2.46

$$a \sin i=31,400,000 \text{ km}$$

$$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.133\ \odot$$

The preliminary and final elements are given in Table III. The probable error of a normal place of weight unity is ±1.33 km/sec. The individual observations are plotted on the velocity-curve in Figure 1.

BOSS 283 = ζ PISCIUM (FT.)

α(1900 $1^h8^m.5$ Spectral type, F5s

δ(1900) $7°3'$ Absolute magnitude, +3.0

The binary character of the fainter component of this well-known double star was announced by J. S. Plaskett in 1919.[3] The star was selected for investigation because the spectrum of the fainter component was suspected on the first spectrograms obtained here. Subsequent plates failed, however, to confirm this suspicion.

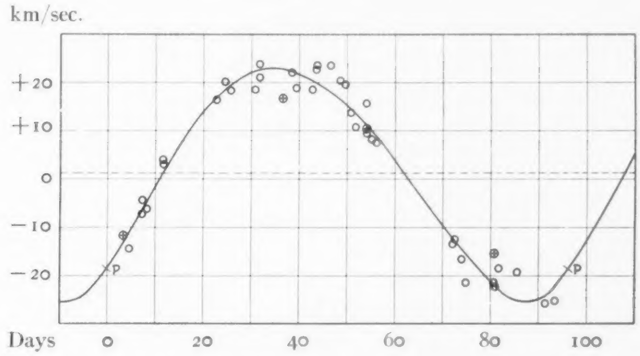

FIG. 1.—Velocity-curve of Boss 9. Crossed circles, 1913–1914 observations

The twenty-seven observations from which the elements were derived are given in Table IV; these were grouped into eleven normal places, weighted according to the number of spectrograms in a group. The normal places are given in Table V.

The four plates taken at Victoria in 1918–1919, which were not included in the solution, serve to establish the period accurately. As some 684 revolutions of the star are covered by the Victoria and the Mount Wilson observations, the adopted period of 9.07504 days may be considered final.

The preliminary elements were found in the manner described for Boss 9, and two least-squares solutions were made in succession for the most probable elements of the orbit. The preliminary and final elements with their probable errors are given in Table VI.

[3] *Journal of the Royal Astronomical Society of Canada*, **13**, 193, 1919.

TABLE IV

OBSERVATIONS OF BOSS 283

Plate	Date	G.M.T.	Phase	Velocity
			days	km/sec.
Victoria........	1918 Oct. 8	20^h47^m	7.246	+46.4
Victoria........	Oct. 27	20 13	8.072	+15.1
Victoria........	Nov. 24	18 37	8.781	− 7.2
Victoria	1919 Jan. 7	16 22	7.311	+43.4
γ 17846........	1930 Nov. 2	21 02	4.786	+45.1
17901........	Nov. 29	16 39	4.379	+34.1
17911........	Nov. 30	17 07	5.398	+57.9
17921........	Dec. 1	17 35	6.418	+53.5
17928........	Dec. 2	17 00	7.382	+45.2
17974........	Dec. 11	15 49	7.269	+49.4
17985........	Dec. 24	17 55	2.206	−25.7
18399........	1931 Aug. 25	23 35	1.418	−41.2
18404........	Aug. 26	23 34	2.417	−29.1
18478........	Oct. 1	19 51	1.961	−39.6
18483........	Oct. 2	19 52	2.962	−14.5
18491........	Oct. 3	19 42	3.955	+19.1
18502........	Oct. 21	19 19	3.789	+13.0
18551........	Nov. 17	18 24	3.526	+ 5.4
18620........	Dec. 29	17 04	0.096	−17.5
18623........	Dec. 30	15 50	1.043	−41.9
18942........	1932 July 16	23 56	0.730	−40.2
18950........	July 17	23 57	1.731	−43.9
19022........	Aug. 17	23 28	5.484	+62.9
19062........	Sept. 8	21 19	0.171	−27.4
19084........	Sept. 11	20 49	3.150	− 7.8
19103........	Sept. 12	20 26	4.135	+21.7
19234........	Nov. 6	18 55	4.620	+32.2
19333........	Dec. 5	16 09	6.280	+58.0
19345........	Dec. 6	15 55	7.270	+43.7
19398........	1933 Jan. 3	16 13	8.061	+16.8
19444........	Jan. 11	15 58	6.972	+50.2

TABLE V

NORMAL PLACES, BOSS 283

Phase	Velocity	Weight	O−C
days	km/sec.		km/sec.
0.139.........	−22.5	½	−0.58
0.887.........	−41.0	½	−2.61
1.573.........	−42.5	½	−1.37
2.200.........	−31.5	¾	+1.78
3.217.........	− 5.6	¾	+0.22
3.965.........	+17.9	¾	−1.12
4.669.........	+37.1	¾	−3.08
5.447.........	+60.4	½	+4.29
6.355.........	+55.8	½	−3.83
7.232.........	+47.1	1	+1.79
8.067.........	+16.8	¼	−1.87

TABLE VI

ORBITAL ELEMENTS, BOSS 283

Preliminary	Final
$P=9.07504$ days	9.07504 days
$K=53.0$ km/sec.	51.07 ± 0.90 km/sec.
$\gamma=+10.0$	$+10.35$
$\omega=135^\circ$	$122^\circ.1\pm7.5$
$e=0.02$	0.033 ± 0.018
$T=$J.D. 4239999.059	J.D. 4239999.162 ± 0.253

$$a \sin i=6,340,000 \text{ km}$$

$$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}=0.124 \odot$$

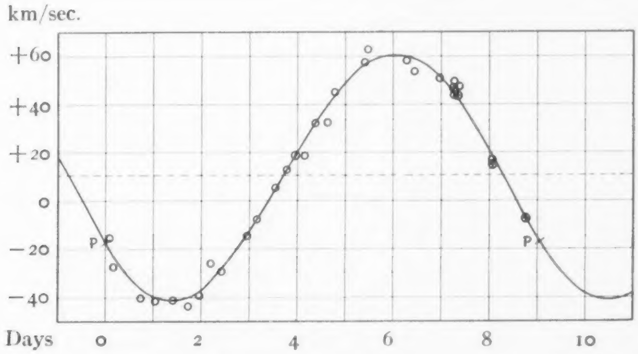

FIG. 2.—Velocity-curve of Boss 283. Crossed circles, Victoria observations

Figure 2 shows the individual observations plotted according to the elements derived above. The sum of the squares of the residuals was reduced from 93.6 to 35.6, or about 62 per cent. The probable error of a normal place of weight unity is ±1.64 km/sec.

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EVAPORATED FILMS FOR LARGE MIRRORS

By ROBLEY C. WILLIAMS and GEORGE B. SABINE

ABSTRACT

An apparatus and technique for depositing metallic films upon large mirrors is described. Chromium and aluminum have been found to possess valuable properties of reflectivity and permanence, and to exhibit no tarnish after several months' use in the laboratory. The reflectivity of aluminum has been observed to decrease from 90 per cent at 6000 A to 80 per cent at 3000 A. Some astronomical advantages of these films are discussed.

Since the process of obtaining metallic films by evaporation was reported by Ritschl[1] in 1931, the technique has been improved and extended to include large surfaces and the use of many metals.[2] Originally the process was used for the deposition of silver on interferometer plates, but later work has indicated the possibility of a wide application to larger surfaces, in particular astronomical reflectors and mirrors for spectrohelioscopes and coelostats.

The apparatus described in this paper is designed for coating surfaces up to 16 inches in diameter in one operation. With slight modification surfaces up to 22 inches in diameter can be coated. The essential parts of the apparatus are a steel bed-plate, a glass bell-jar, and a high-vacuum pumping system.

The bed-plate is of cold-rolled steel, 22 inches square and 1 inch thick. Into twelve tapered holes in the plate are sealed glass plugs each containing three 50-mil tungsten leads. These holes are arranged in two concentric circles, the outer circle containing eight holes and the inner one four. To each of the tungsten leads is attached one end of a filament to be heated, the other end being connected through a conveniently placed brass post to the bed-plate. Thus the electrical circuit for heating the filament consists of an external control resistance, the tungsten lead, the filament, and the bed-plate, which is a common terminal for all the filaments. In case the metal to be evaporated has a melting point below 2000° C., it can be inserted in any convenient form into the filament, which is

[1] *Zeitschrift für Physik*, **69**, 578–585, 1931.

[2] C. H. Cartwright, *Review of Scientific Instruments*, **3**, 298–304, 1932; R. C. Williams, *Physical Review*, **41**, 255, 1932.

a small helix of 10-mil tungsten wire. For metals having high melting points, in particular chromium, it is necessary to deposit the metal electrolytically upon the filament.

The surface to be coated is supported at its periphery by three brass rods of adjustable length screwed into the bed-plate. The optimum distance from the surface to the filaments is about 10 cm for a pressure of 10^{-4} mm of mercury, which distance is well below the mean free path for this pressure.

The bottom of the Pyrex bell-jar is ground smooth, and with the aid of low vapor-pressure wax, an air-tight joint is obtained. To protect the wax sucked under the seal from bombardment by the filaments, a hoop of brass is fitted to the inside of the bell-jar. The bell-jar is evacuated through an outlet in the bed-plate by means of an oil fore-pump and a two-stage mercury diffusion pump. It has been found that bright films cannot be obtained with an air pressure of more than 10^{-4} mm of mercury.

The process of coating a 16-inch mirror requires about fifteen minutes after the proper vacuum is obtained. The filaments are heated one at a time, and each filament coats with sufficient density about 20 square inches of the surface. Some metals, particularly chromium, cannot be removed from glass by ordinary means: In case removal of the chromium is contemplated, a thin layer of gold is first evaporated upon the surface, one of the three filaments in each unit being used for this purpose. For the reflecting surface of chromium either of the two remaining filaments can be used, the other serving as a reserve filament in case the first is accidentally burned out.

Chromium and aluminum were found to possess properties of reflectivity and permanence that silver lacks.

Chromium.—Applied directly to very clean glass or placed over a layer of gold, chromium exhibits no tarnish whatsoever on prolonged exposure to ordinary atmospheric conditions, or even to atmospheres of ozone, hydrogen sulphide, and sulphur dioxide. With some kinds of soft glass only abrasives or hydrofluoric acid are able to remove a chromium coat applied directly. There is an optimum thickness for the chromium film, namely, that beyond which the reflectivity does not increase with thickness. Beyond this thickness the film tends to

peel off, and in so doing actually disfigures the underlying glass surface. A film of optimum density is not nearly opaque, but transmits about 25 per cent of the incident light, and reflects about 65 per cent. By our measurements the reflectivity of evaporated films of chromium is 60 per cent at 4200 A, 68 per cent at 3450 A, and 62 per cent at 2900 A.

Aluminum films do not have the mechanical durability of chromium, and can be removed by hydrochloric acid. However, they resist tarnish exceptionally well, as is shown by the fact that no observable deterioration of the surface resulted from several days' exposure to atmospheres of pure moist oxygen and sulphur vapor. The degree of cleanliness of the glass determines whether or not the film will acquire a "frosted" appearance upon exposure to air. Clean surfaces coated in our laboratory after several months' exposure to air have no trace of the "frosted" appearance characteristic of exposed massive blocks. Aluminum possesses remarkable reflective power over the entire range of the spectrum tested. The reflectivity decreases almost uniformly from 90 per cent at 6000 A to 73 per cent at 2300 A. An aluminum surface that has been in use for three months shows no measurable decrease in reflectivity. The reflectivity at 3000 A, the limit of transmission through the earth's atmosphere, is 80 per cent. It is evident that owing to its permanence and high reflectivity in the visible and near ultra-violet regions of the spectrum, aluminum is peculiarly suitable for use in astronomical reflectors. Furthermore, owing to its low melting point it is readily evaporated.

In order to give a definite idea of the relative efficiency of aluminum and silver for use in reflecting telescopes, the curves in Figure 1 have been drawn showing the ratio of the intensity of light reflected at various wave-lengths from new silver to that reflected from aluminum, for the case of one, two, and three reflections. The ratio is obtained from the data on *new* silver in the *International Critical Tables*, and from our data on aluminum. Of course, silver that has been in use for a few weeks suffers a considerable decrease in reflectivity, particularly from 4000 to 3000 A. The curves may be taken to show the relative times of exposure necessary to obtain the same photographic density at any wave-length, for aluminum and silver

reflectors. The reflectivity ratio of three silver to three aluminum mirrors is shown in curve *III;* of two silver to two aluminum mirrors in curve *II.* Curve *II* likewise shows the reflectivity ratio of three silver mirrors to a combination of one silver and two aluminum. Thus it is seen that a large increase in efficiency for wave-lengths shorter than 3500 A would be obtained in a three-mirror instrument by using aluminum instead of silver as the reflecting surface for the two auxiliary mirrors. The relative efficiency of this combination as compared with three aluminum mirrors is shown in curve *I.* Thus, although aluminum for all three reflecting surfaces is far superior for ultra-violet spectra, the combination of two aluminum mirrors and

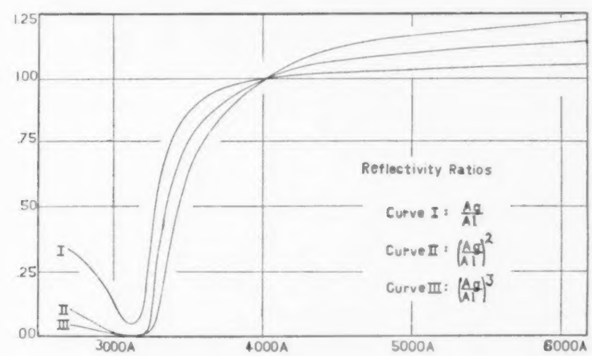

FIG. 1.—Reflectivity ratios for silver and aluminum

one silver mirror will reflect over 65 per cent as much ultra-violet light as does a single silver mirror. Accordingly, a three-mirror telescope with its accompanying mechanical advantages could be used satisfactorily over the entire astronomical spectrum, even when the large concave mirror is silvered. As a numerical example of the relative efficiencies of silver and aluminum, let us assume that a spectrum at 3250 A could be obtained in one hour with three newly silvered mirrors. With three aluminum mirrors, the exposure time would be about one and one-half minutes; with two aluminum mirrors and one silver mirror, the time would be about four minutes.

To test the astronomical possibilities of a chromium surface, a spectrum of α Lyrae was taken under ordinary atmospheric conditions at the Fuertes Observatory. A quartz spectrograph was used

at the principal focus of a 10-inch chromium-coated glass mirror. Figure 2 is a microphotometer record of the spectrum obtained. It is to be noted that the visible region did not have to be heavily exposed in order to obtain a usable exposure from 3600 to 3100 A. The curve further demonstrates that an unusually clear atmosphere is not necessary for obtaining the ultra-violet spectra of stars when a suitable reflector is used. An aluminum mirror would have given even better results.

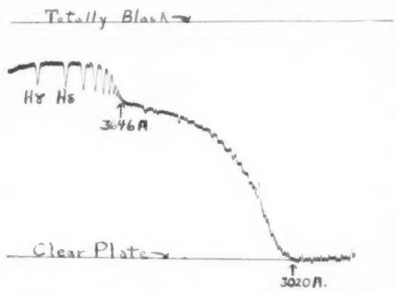

FIG. 2.—Microphotometer record of the spectrum of α Lyrae.

The thinness and homogeneity of the films has been well demonstrated by evaporating both aluminum and chromium upon glass diffraction gratings. In no case was the quality of their spectra impaired by the deposition of such films upon the gratings, and a great increase in intensity in the visible and near ultra-violet was noted. A glass grating with thirty thousand lines per inch was coated with chromium and placed in a vacuum spectrograph for use in the extreme ultra-violet. The intensity of lines in the neighborhood of 200 A was enhanced, while the general appearance of the spectrum was not notably changed by the addition of the chromium film to the grating.

The authors wish to express their thanks to Professor S. L. Boothroyd for his suggestions and advice during the progress of this investigation.

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THE SPECTRUM OF THE Bo STAR 23 τ SCORPII

By O. STRUVE AND T. DUNHAM, JR.

ABSTRACT

Wave-lengths of 296 absorption lines in the spectrum of the Bo star τ Scorpii are given. Of these 243 have been identified with *H*, *He* I, *He* II, *C* II, *C* III, *N* II, *N* III, *O* II, *O* III, *Mg* II, *Al* III, *Si* III, *Si* IV, *P* III, *S* II, *S* III. The widths of helium lines which do not belong to the diffuse series and which are immune to electrical fields have been tested for thermal Doppler broadening. *He* 3965 has a core 0.55 A wide, in agreement with theory, which predicts that it should be between 0.4 and 1.0 A.

There are surprisingly few stars of spectral class earlier than B2 in which the lines are not diffuse when photographed with spectrographs of even moderate dispersion. The spectrum of τ Scorpii, class Bo, is exceptional in this respect; its lines are perfectly sharp and narrow and there is not the least trace of what has been referred to as rotational broadening. A spectrogram of small dispersion of τ Scorpii was measured by Struve in his paper on B stars.[1] In a later paper[2] Struve suggested that it would be of interest to measure the widths of such helium lines as are not appreciably disturbed by electric fields. A preliminary computation indicated that the core of such a line should have a width of approximately 1 A, provided the atoms move at random with an average velocity corresponding to a temperature of 24,000 K.

A spectrogram of τ Scorpii has recently been obtained by Dunham with the coudé spectrograph and the 100-inch Mount Wilson reflector. The linear dispersion of the plate is about 12 A per millimeter at λ 4500, and the measurable range of the star's spectrum is from about λ 3945 to λ 4713. Eastman Process emulsion was used to obtain better contrast.

The wave-lengths and identifications are given in Table I. Table II contains a summary of the lines for each element. It will be seen that *O* II is especially well represented: In the middle of our plate, where photographic density and focus are best, all of the lines measured by A. Fowler[3] in the laboratory have been identified. In fact,

[1] *Astrophysical Journal*, **74**, 225, 1931.

[2] *Proceedings of the National Academy of Sciences*, **18**, 585, 1932.

[3] *Proceedings of the Royal Society*, A, **110**, 476, 1926.

TABLE I

WAVE-LENGTHS IN τ SCORPII

Observed Wave-Length	Int.	Identification	Notes
3945.04	5	*O* II 5.05 (5)	
3954.36	7	*O* II 4.37 (7)	
3955.85	1	*N* II 5.85 (6)	
3961.60	2	*O* III 1.59 (8); *S* III 1.56 (3)	
3963.0	1	*O* II 3.13 (0)	
3964.78	8	*He* I 4.73 (4)	
3970.02	2	$H\epsilon$ 0.08	1
3973.27	3	*O* II 3.27 (10)	
3982.74	4	*O* II 2.72 (5)	
3983.62	1	*S* III 3.75 (4)	
3995.03	10	*N* II 5.00 (10)	
3998.65	1	*N* III 8.69 (3); *S* II 8.77 (3)	
4003.58	2	*N* III 3.64 (4)	1
4009.42	4	*He* I 9.27; [*C* II 9.90 (2)]	1
4025.52	1	Forbidden *He* I 5.48	
4026.31	30	*He* I 6.19 (5); *He* I 6.36 (1); *N* II 6.09 (3)	1
4027.33	1	....	
4035.10	3	*N* II 5.09 (4); *O* II 5.00 (0)	
4041.37	4	*N* II 1.32 (5); *O* II 1.20 (0)	
4041.96	1	....	
4042.49	1	....	
4043.68	3	*N* II 3.54 (3)	
4048.23	1	*O* II 8.22 (1)	1
4054.23	1	*O* II 4.60 (0)	
4056.09	3	*C* III 6.06(4)	
4060.81	3	*O* II 0.58 (3); *O* II 0.98 (2)	3
4062.17	1	....	
4063.02	2	*O* II 2.90 (1)	
4064.25	1	*S* III 4.40 (3)	1
4066.54	1	....	
4067.31	1	....	
4067.95	5	*C* III 7.87 (6)	
4068.91	4	[*C* II 8.97 (3)]; *C* III 8.94 (7)	
4069.72	7	*O* II 9.90 (6); *O* II 9.64 (4)	
4070.24	3	[*C* II 0.30 (3)]; *C* III 0.43 (8)	
4071.27	3	*O* II 1.20 (0)	
4072.16	6	*O* II 2.16 (8)	
4072.96	1	*N* II 3.04 (2)	
4074.3	1	*O* III 3.90 (0)	
4074.75	1	*C* II 4.53 (2); *C* II 4.89 (1)	1
4075.90	8	*O* II 5.87 (10)	
4076.89	1	[*N* II 6.83 (0)]	
4077.72	1	....	2
4078.84	3	*O* II 8.86 (4); *O* II 9.00 (0)	
4081.1	1	*O* III 1.10 (1)	5?
4082.31	1	*N* II 2.28 (2)	
4083.89	3	*O* II 3.90 (2)	
4084.60	1	*O* II 4.66 (1)	
4085.20	3	*O* II 5.12 (3)	
4086.78	3	*O* II 7.16 (2)	
4087.78	1	[*N* II 7.35 (0)]	2
4088.87	10	*Si* IV 8.86 (10)	

TABLE I—*Continued*

Observed Wave-Length	Int.	Identification	Notes
4089.29	3	*O* II 9.28 (4)	
4092.97	2	*O* II 2.94 (5)	
4095.66	2	*O* II 5.63 (0)	
4097.31	6	*O* II 7.25 (4); *N* III 7.31 (10)	
4101.71	30	*H*δ 1.74	1
4103.42	3	*N* III 3.37 (9)	
4105.02	3	*O* II 5.00 (7); *O* II 4.75 (5)	
4106.10	1	*O* II 6.03 (0)	2
4107.19	1	*O* II 7.07 (1)	2
4108.0	1	*O* II 8.75 (0)	2
4110.79	3	*O* II 0.80 (3); *O* II 0.20 (1)	
4112.07	3	*O* II 2.04 (4)	
4113.89	1	*O* II 3.82 (1)	2
4116.16	10	*Si* IV 6.10 (8)	
4119.25	7	*O* II 9.22 (8)	
4120.34	2	*O* II 0.30 (3); *O* II 0.55 (2)	
4120.87	8	*He* I 0.81 (3); *He* I 0.98 (1)	1
4121.64	1	*O* II 1.48 (4); *C* III 2.05 (3)	
4128.62	1	....	
4129.35	1	*O* II 9.34 (2)	
4132.79	5	*O* II 2.82 (6)	
4140.6	1	*O* II 0.74 (0)	
4142.09	1	*O* II 2.00 (1); *S* II 2.31 (4); *O* II 2.24 (0)	
4143.80	10	*He* I 3.77 (2); *O* II 3.77 (2); *O* II 3.52 (1)	1
4144.74	1	*S* II 5.05 (8)	
4146.00	2	*O* II 6.09 (3); *N* II 5.76 (3); *O* II 5.90 (0)	
4147.14	1	*S* II 6.90 (5)	
4152.54	2	*C* III 2.43 (2)	
4153.33	5	*O* II 3.31 (7); *S* II 3.14 (6)	
4154.34	1	....	
4156.54	3	*O* II 6.54 (3); *C* III 6.50 (3)	
4162.91	3	*C* III 2.80 (4)	
4164.81	3	[*S* II 4.98 (2)]	
4166.70	1	....	1
4168.01	1	[*N* II 7.50 (0)]; *S* II 8.37 (5)	2
4169.12	6	*He* I 8.97 (1); *O* II 9.23 (4)	1
4171.60	1	*N* II 1.63 (2)	
4176.22	3	*N* II 6.17 (3)	1
4179.75	1	*N* II 9.68 (1)	
4181.3	1	*N* II 0.89 (0); *N* II 1.17 (0)	
4185.49	4	*O* II 5.45 (8)	
4186.94	5	*C* III 7.05 (10)	
4188.17	1	[*Al* III 8.88 (5)]	1
4189.86	6	*O* II 9.79 (10)	
4192.42	1	*O* II 2.50 (2)	
4195.79	2	*N* III 5.70 (5)	
4196.79	1	*O* II 6.72 (1)	2
4200.16	3	*N* III 0.02 (6); *He* II 9.87	
4206.56	1	*N* II 7.51 (1); *N* II 6.57 (0); *N* II 6.35 (0)	2
4212.48	4	*Si* IV 2.44 (3)	
4215.79	1	[*N* II 5.72 (0)]	
4217.22	2	*S* II 7.19 (4)	
4218.28	1	....	
4219.64	3	....	

TABLE I—*Continued*

Observed Wave-Length	Int.	Identification	Notes
4222.04	1	*P* III 2.15 (7)	
4227.68	2	*N* II 7.83 (3)	
4228.92	1	....	
4231.70	2	....	
4235.08	1	....	1
4236.95	4	*N* II 6.98 (6)	
4241.80	4	*N* II 1.80 (8)	1
4249.83	1	....	2
4250.82	1	....	2
4253.81	12	*O* II 3.98 (8); *S III* 3.51 (6)	4
4256.88	1	....	2
4263.67	1	....	2
4265.83	2	....	
4267.19	9	*C* II 7.27 (10); *C* II 7.02 (8)	
4270.50	1	....	2
4273.00	1	*O* II 3.17 (0)	2
4275.60	4	*O* II 5.52 (4)	
4276.62	2	*O* II 6.71 (1)	1
4277.73	2	*O* II 7.40 (1); *O* II 7.90 (1)	1
4279.19	1	*S* II 8.62 (3)	
4281.41	2	*O* II 1.40 (0)	
4283.00	2	*O* II 2.96 (1)	
4283.79	2	*O* II 3.75 (0)	
4284.97	3	*S* III 5.00 (5); *N* III 4.51 (1)	
4285.77	2	*O* II 5.70 (3)	
4288.90	3	*O* II 8.83 (1); *N* III 8.72 (1)	
4290.53	3	*N* III 0.55 (1); *N* III 0.80 (3)	
4291.34	3	*O* II 1.25 (1)	
4292.25	1	*O* II 2.23 (0)	
4294.82	4	*O* II 4.82 (3); *N* III 4.76 (0)	
4300.64	1	....	2
4302.91	2	*O* II 3.00 (0)	
4303.85	7	*O* II 3.82 (5)	
4305.56	1	*O* II 5.53 (0)	2
4307.20	2	*O* II 7.31 (1)	
4309.07	2	*O* II 8.96 (1)	
4310.39	1	....	
4312.06	2	*O* II 2.10 (0)	
4313.52	2	*O* II 3.43 (1)	
4314.02	1	....	
4315.58	2	*O* II 5.55 (0)	1
4317.16	6	*O* II 7.16 (8)	
4317.76	1	....	
4319.64	7	*O* II 9.65 (8); *O* II 9.93 (1)	
4325.63	5	*O* II 5.77 (3); *C* III 5.70 (7)	
4327.58	3	*O* II 7.48 (3); *O* II 7.89 (0)	
4328.31	2	*O* II 8.62 (2); *N* III 8.15 (3)	
4329.59	1	*N* III 0.14 (2)	2
4331.13	1	*O* II 1.89 (2); *O* II 1.47 (0); *O* II 1.21 (0)	2
4332.76	2	*O* II 2.76 (1); *S* III 2.69 (6)	
4336.90	3	*O* II 6.86 (6)	
4340.47	40	$H\gamma$ 0.47	1
4343.47	1	*O* II 3.30 (0)	
4344.46	1	*O* II 4.42 (0)	

TABLE I—*Continued*

Observed Wave-Length	Int.	Identification	Notes
4345.60	10	*O* II 5.57 (7)	
4347.47	8	*O* II 7.43 (5)	
4349.43	12	*O* II 9.44 (8)	
4351.30	8	*O* II 1.28 (6)	
4353.61	1	*O* II 3.60 (1); *N* III 3.66 (2)	
4354.66	1	*S* III 4.58 (5)	
4355.70	1	....	2
4357.09	2	*O* II 7.25 (0)	
4358.95	1	*O* II 8.40 (0); *O* II 9.38 (1)	
4361.62	3	*S* III 1.57 (6); *C* III 1.85 (2)	
4364.86	2	*S* III 4.77 (4); [*Al* III 4.59 (2)]	
4366.94	8	*O* II 6.91 (7)	
4368.23	1	*C* III 8.14 (4)	
4369.33	1	*O* II 9.28 (4)	
4371.69	1	*O* II 1.65 (2)	
4372.69	1	[*C* II 2.49 (1)]	
4374.64	1	*C* II 4.28 (2); *N* II 5.00 (0)	
4378.20	3	*O* II 8.40 (3)	4
4379.43	3	*N* III 9.09 (10); *O* III 9.55 (3)	4
4387.30	1	....	
4388.04	20	*He* I 7.93 (3)	1
4390.20	1	*Mg* II 0.58 (10)	2
4391.96	3	[*S* II 1.94 (1)]	
4393.27	1	....	
4394.52	1	....	
4395.99	3	*O* II 5.95 (7)	
4396.71	1	....	2
4398.01	2	....	
4399.35	1	[*Ca* III 9.64 (10)]	
4405.96	1	*O* II 6.02 (1)	
4408.8	1	*O* III 8.14 (1)	2
4409.27	3	[*C* II 0.06 (1)]	
4411.22	2	*C* II 1.20 (2); *C* II 1.52 (2)	
4413.35	2	....	
4414.96	10	*O* II 4.89 (10)	
4415.69	2	....	
4417.05	10	*O* II 6.97 (8)	
4419.97	1	....	
4428.53	3	[*N* II 7.97 (2)]	
4429.84	2	[*O* III 0.2 (0)]	
4430.99	2	[*S* II 1.14 (1)]	
4432.67	1	*N* II 2.71 (6); *S* II 2.45 (2)	
4433.47	1	*N* II 3.48 (2); *Mg* II 3.99 (8)	2
4434.6	1	*O* III 4.43 (2)	2
4435.92	1	[*O* III 5.4 (0)]	
4437.60	5	*He* I 7.55 (1)	1
4439.11	1	....	
4439.87	1	*O* III 0.1 (0); *S* III 9.88 (4)	
4441.89	1	*N* II 1.99 (3)	
4442.95	2	*O* II 3.05 (5)	
4445.44	1	....	
4446.97	3	*N* II 7.04 (10); *O* II 7.08 (1)	
4448.28	3	*O* II 8.21 (6)	
4450.54	1	....	

TABLE I—*Continued*

Observed Wave-Length	Int.	Identification	Notes
4452.40	2	*O* II 2.38 (6)	
4454.08	1	*O* III 4.00 (1)	
4457.01	1	[*S* II 6.39 (4)]	
4458.00	1	*O* III 8.44 (1)	
4465.36	3	*O* II 5.40 (4)	
4466.41	2	*O* II 6.32 (2)	
4467.91	2	*O* II 7.88 (4); *O* II 7.55 (1)	
4469.75	6	*O* II 9.32 (3); forbidden *He* I 9.97	1
4471.56	20	*He* I 1.48 (6); *He* I 1.69 (1)	1
4477.92	1	*O* II 7.88 (2); *N* II 7.74 (2)	2
4479.97	1	*Al* III 9.97 (4); *Al* III 9.89 (3)	
4481.21	8	*Mg* II 1.33; *Mg* II 1.13	
4482.98	1	*O* III 2.78 (1)	
4483.93	1	*S* II 3.48 (1)	
4487.8	1	*O* II 7.72 (0); *O* II 8.09 (2)	4
4491.24	1	*O* II 1.25 (3)	
4493.85	1	....	2
4495.62	1	....	2
4496.17	1	....	
4499.4	1	....	
4504.18	1	....	2
4506.3	1	*O* II 6.50 (2)	2
4507.6	1	*N* II 7.58 (3)	
4510.88	3	*N* III 0.92 (6)	
4512.54	1	*Al* III 2.54 (4)	
4514.98	3	*N* III 4.89 (7)	
4516.74	1	*C* III 6.69 (3)	
4518.20	1	*N* III 8.18 (3)	2
4523.40	2	*N* III 3.60 (4)	
4529.21	2	*Al* III 9.18 (6); *Al* III 8.91 (1)	
4530.40	1	*N* II 0.37 (5)	
4534.68	1	*N* III 4.57 (3); *N* III 5.11 (2)	3
4540.9	1	*He* II 1.63	3
4544.9	1	....	
4546.6	1	*N* III 6.36 (3)	
4549.3	1	....	
4552.64	7	*Si* III 2.65 (9); *N* II 2.50 (4)	
4567.84	7	*Si* III 7.87 (7)	
4570.55	1	[*O* III 9.50 (1)]	2
4573.05	1	....	
4574.83	6	*Si* III 4.78 (4)	
4590.97	5	*O* II 0.98 (9)	
4592.19	1	....	2
4593.38	1	*C* III 3.47 (1)	2
4596.27	5	*O* II 6.19 (8)	
4601.84	2	*N* II 1.49 (8); *O* II 2.11 (2)	1
4605.60	1	....	
4607.37	2	*N* II 7.17 (7)	
4609.43	3	*O* II 9.42 (4)	
4610.27	2	*O* II 0.14 (3)	
4611.94	1	....	
4613.84	2	*N* II 3.88 (6)	
4621.49	2	*N* II 1.40 (7)	
4628.11	2	....	1

TABLE I—*Continued*

Observed Wave-Length	Int.	Identification	Notes
4630.93	5	*N* II 0.55 (10); *Si* IV 1.38 (3)	1
4634.26	3	*N* III 4.16 (8)	
4636.59	1	....	
4638.88	3	*O* II 8.86 (6)	
4640.71	3	*N* III 0.64 (10)	
4641.96	4	*O* II 1.83 (9); *N* III 1.90 (3)	
4643.19	1	*N* II 3.11 (8)	
4644.69	1	....	
4647.49	9	*C* III 7.40 (10)	
4649.24	6	*O* II 9.15 (10)	
4650.49	5	*C* III 0.16 (9); *O* II 0.85 (6)	
4651.50	5	*C* III 1.35 (8)	
4654.53	4	*Si* IV 4.14 (4); *N* II 4.57 (2)	
4659.06	1	*N* III 8.88 (3)	
4661.84	4	*O* II 1.65 (9)	
4663.82	1	*C* III 3.53 (3)	
4665.93	2	*C* III 5.90 (5); *Si* III 5.87 (0)	
4667.44	1	*N* III 7.18 (5); *N* II 7.28 (2)	
4673.84	3	*O* II 3.75 (4); *C* III 3.91 (4)	4
4676.25	3	*O* II 6.25 (8)	4
4685.78	8	*He* II 5.81	1
4689.19	1	....	
4699.13	3	*O* II 9.21 (7)	
4703.13	1	*O* II 3.18 (3)	
4705.57	3	*O* II 5.36 (8)	
4707.73	1	....	
4710.07	1	*O* II 0.04 (5)	
4713.29	8	*He* I 3.14 (3); *He* I 3.37 (1)	

NOTES TO TABLE I

(1) Diffuse; (2) uncertain; (3) blend; (4) broad; (5) double.

the limit of visibility for *O* II seems to be about the same on our plate of τ Scorpii as on the laboratory plate used by Fowler. It is possible, however, that a few fainter lines could have been identified in the star, if their laboratory wave-lengths had been available.

Many new identifications have been made for *C* III, *N* III, and *O* III, but Table I contains no elements other than those listed by Struve.[4]

Table III lists the neutral helium lines which have been measured on our plate. For each line we have given the shift in angstroms produced by an electrical field of 100 kv. These data have been taken from the diagrams of Y. Ishida and G. Kamijima.[5] All members of the two diffuse series (singlets and triplets) split up into many com-

[4] *Astrophysical Journal*, p. 241.

[5] *Scientific Papers of the Institute of Physical and Chemical Research*, **9**, 117, 1928.

ponents and are therefore unsuitable for a test of thermal Doppler broadening. These lines are diffuse in the spectra of stars having dwarf characteristics. Some of the other lines—for example, those belonging to series $2S-mP$ (λ 3965); $2p^3-ms^3$ (λ 4121, 4713); and $2P-mS$ (λ 4438)—are practically immune to electrical fields not exceeding 100 kv. The ionic fields in stellar atmospheres practically never approach this value. The average intensity of the field in a stellar atmosphere is from 1 to 10 kv., and the frequency of more intense fields declines quite rapidly.[6] In any case, these lines of *He* I

TABLE II

NUMBER OF IDENTIFIED LINES

Element	Group I	Group II	Group III	Element	Group I	Group II	Group III
H	3	....	....	*Mg* II	1	1	2
He I	11	4	1	*Al* III	3	2	2
He II	2	....	1	*Si* III	3	....	1
C II	4	3	5	*Si* IV	4	....	1
C III	12	....	8	*P* III	1	....	....
N II	25	1	16	*S* II	5	....	9
N III	16	2	9	*S* III	6	....	4
O II	105	15	18	*Ca* III	....	....	1
O III	7	....	6				

Group I contains definitive identifications.

Group II contains weaker components of blends in which the stronger components belong to the same atom.

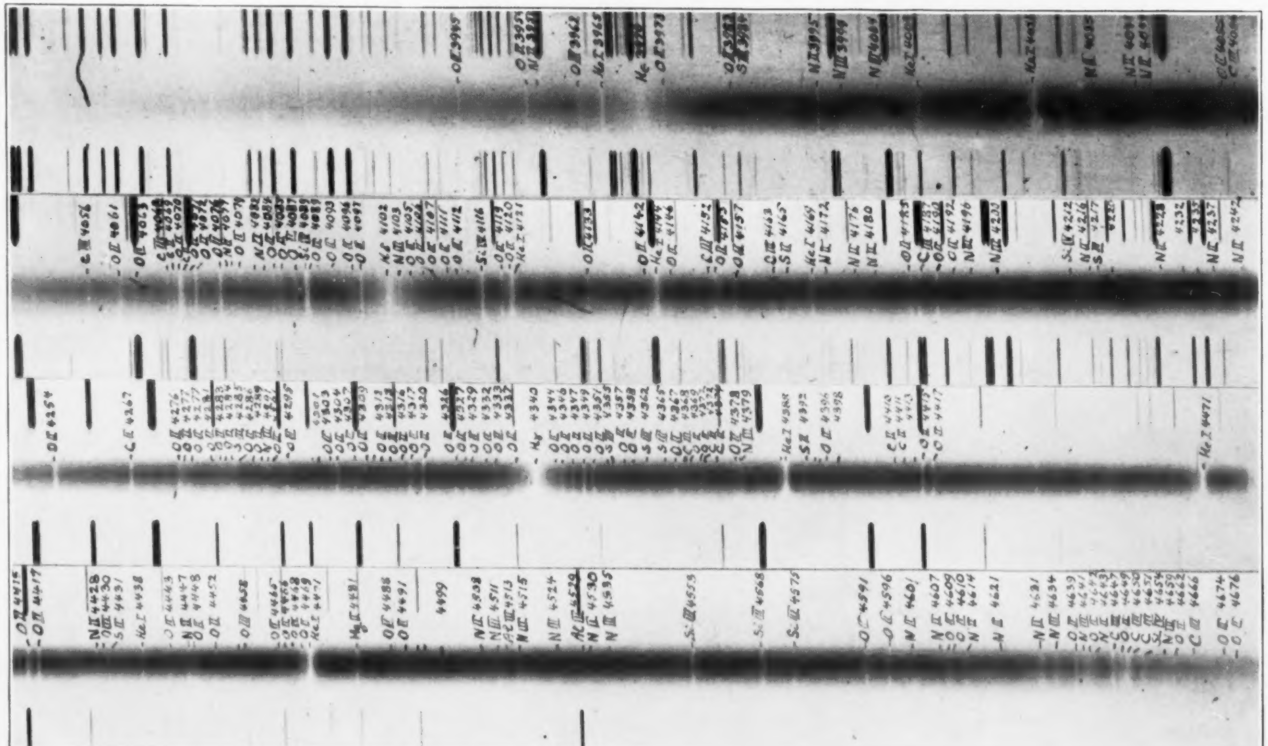
Group III contains weaker components of other blends, and uncertain identifications.

are as little affected by Stark broadening as are the lines of other, heavier elements, and it would therefore seem that they would be especially suitable for testing the effect of thermal Doppler broadening. The line λ 4169 ($2P-6S$) is less suitable because it has an appreciable positive shift in the electric field.

Of the lines mentioned above, λ 4121 and λ 4169 are appreciably broader than are lines of other elements. But λ 4121 lies between two rather strong lines, 4120.34 [*O* II 0.30 (3); *O* II 0.55 (2)] and 4121.64 [*O* II 1.48 (4); *C* III 2.05 (3)], and may be partly blended with these. *He* 4169 is blended with *O* II 4169.23 (4) and would therefore naturally have a broadened contour, even if it were not itself subject to Stark effect. *He* 3965 and *He* 4438 are perhaps very slightly broader than are neighboring lines of *O* II or of other ele-

[6] Struve, *Astrophysical Journal*, **69**, 173, 1929; **70**, 85 and 237, 1929.

PLATE XII



THE SPECTRUM OF τ SCORPII (B0)

ments which are of about the same intensity. This may be due to thermal broadening. Actual measurement of the width of the core of *He* 3965 gives 0.55 A. The slit width used for our exposure amounted to about 0.27 A at λ 3965, consequently the width of the core is actually less than 0.55 A.

According to Pannekoek,[7] the width of the core depends to some extent upon the total intensity of the line, and may, for weaker lines, attain a value of approximately one-half that of moderately strong lines. The criterion is the relative intensity of the light in the Dop-

TABLE III

HELIUM LINES IN τ SCORPII

Wave-Length	Series Designation	Shift at 100 Kv.		Notes
		p	s	
3965	$2S-4P$	0 A	0 A	
4009	$2P-7D$	....	....	*
4024	$2P-7S$	....	....	
4026	$2p^3-5d^3$	+2	0	*
4121	$2p^3-5s^3$	0	0	
4144	$2P-6D$	7	+5	*
4169	$2P-6S$	4	4	
4388	$2P-5D$	4	4	*
4438	$2P-5S$	1	0	
4470	$2p^3-4f^3$	....	....	*
4472	$2p^3-4d^3$	+3	+2	*
4713	$2p^3-4s^3$	0	0	

* Splits up into many components.

pler core and in the outer wing produced by radiation damping. *He* 3965 is not strong enough to show the radiation-damping wings. Consequently, for atomic weight 4 and temperature 24,000 K, the core should not be wider than 1.0 A. On the other hand, the central intensity of this line is small, approximately 0.2 of that of the continuous spectrum. Consequently, it would follow from the computations and diagrams of Pannekoek that the width of the core should not be less than 0.4 A. The measured value is thus within the range predicted by theory. It will be of interest to measure the width of λ 3965 on spectrograms of greater dispersion. The linear dispersion of our original plate at λ 3965 is 7.8 A/mm.

YERKES OBSERVATORY
MOUNT WILSON OBSERVATORY
January 17, 1933

[7] *Monthly Notices of the Royal Astronomical Society*, **91**, 157, 1930.

SOME EVIDENCE FOR THE EXISTENCE OF A PECULIAR BRANCH OF THE SPECTRAL SEQUENCE IN THE INTERVAL B8–F0

By W. W. MORGAN

ABSTRACT

The peculiar spectra (exclusive of the *c*-stars) in the range B8–Fo can be arranged in an apparent temperature sequence which may be discrete from the normal giants and dwarfs. Each group of peculiar spectra is related to the groups of next higher and lower effective excitation. In order of decreasing degree of ionization the groups and type stars are: the "*Mn* II stars" (α Andromedae and μ Leporis), stars in which the unidentified line at λ 4200 is well marked (θ Aurigae), the *Eu* II stars (α^2 Canum Venaticorum at maximum of *Eu* II), the *Cr* II stars (73 Draconis), and the "*Sr* II stars" (γ Equulei). All of these groups overlap the contiguous ones. If the peculiar stars brighter than 5.5 mag. are arranged in these groups, it is found that the peculiar branch contains all the silicon stars.

The variations in the spectrum of α^2 Canum Venaticorum can be interpreted in terms of such a peculiar sequence as being due to changes in effective excitation similar to those observed in Cepheids in the normal sequence.

The peculiar A-type spectra were grouped by the *Henry Draper Catalogue* into the following classes: the *c*-stars, which are now known to be stars of high luminosity, the silicon stars, the strontium stars, stars resembling α Andromedae, stars resembling θ Aurigae, and stars resembling α^2 Canum Venaticorum. Considerably later, Adams and Joy[1] called attention to several A-type strontium stars in which *Cr* II is unusually strong. The first of these groups has been rather completely investigated and will not be considered. Since the classification of the stars in the *Henry Draper Catalogue* was completed, a number of new members of the peculiar groups have been found and it now seems worth while to examine the peculiar spectra for evidence of their relation to one another and to normal stars of the same spectral types. An additional group in which the unidentified line at λ 4200 is well marked has been introduced.

The plates used are of one-prism dispersion, having a scale of 30 A per millimeter at λ 4500. The spectrograms were obtained with the Bruce spectrograph attached to the forty-inch telescope of the Yerkes Observatory. All the standard stars had been photographed on the fine-grain Eastman Process emulsion, with the exception of the Fo

[1] *Publications of the Astronomical Society of the Pacific*, **38**, 124, 1926.

stars γ Equulei and μ Ceti, which had been taken on Eastman-40 plates. The use of the Process plates made all the lines appear much stronger than on the Eastman-40 emulsion and brought into view many lines which were entirely invisible on the plates of coarser grain.

An examination of the peculiar spectra showed that the five groups could be arranged in order of effective excitation, the manganese stars being the hottest and the chromium and strontium stars the coolest. The characteristic of abnormal strength in the *Si* II doublet λ 4128 and λ 4131 was not considered in this arrangement and will be taken up later. Type stars were selected for each of the five groups. These stars are listed in Table I, which gives the name of

TABLE I

Star	Spec.	Group	Spectral Range
α Andromedae	B8	Manganese	B8–A0
μ Leporis	B9		
θ Aurigae	B9–A0	λ 4200	B9–A0
α^2 Canum Venaticorum (max. *Eu* II)	A0	Europium	B9–F0
73 Draconis	A2	Chromium	A0–F0
γ Equulei	F0	Strontium	A2–F0

the star, its revised spectral type, the group which it represents, and the range of spectrum over which the group is observed.

For comparison with these type stars the following dwarfs were taken as standards for their spectral subdivisions: B8, 20 Tauri (Maia); B9, κ Cephei; A0, Sirius; A2, ϵ Serpentis; F0, μ Ceti. The principal differences between the normal and the peculiar stars are shown in the following comparison.

α Andromedae and Maia.—The *He* I lines are very slightly stronger in Maia than in α Andromedae. The metallic lines *Fe* II 4233 and *Fe* II–*Ti* II 4549 are also slightly stronger in Maia. *C* II 4267, the *Si* II doublet λ 4128 and λ 4131, and *Ca* II K are all of approximately the same intensity in the two stars. From these similarities the stars are shown to be of about the same spectral type. At this stage the intensities of the helium lines and of *C* II 4267 increase rapidly with increasing temperature, while such lines as *Fe* II–*Ti* II 4549 rapidly decrease; these lines therefore form sensitive criteria for classifica-

tion. On the basis of the classification with Maia the spectrum of α Andromedae would be classed as B8, and there seems to be no detail of the spectrum which would call for a classification later than B9. In the range $H\beta$–$H\epsilon$ there are no neutral metallic lines (the line near λ 4045 is considerably displaced toward the violet of the position of the strong *Fe* I laboratory line and cannot be due to neutral iron) and only a very few of the strongest lines of *Fe* II are present. The actual number of lines present in the spectrum, however, is as great as in many stars of type A0. Most of these lines are due to *Mn* II, of which there does not seem to be a trace in Maia. There are also some well-marked lines for which no identification can be found. There is a faint line at λ 4200 agreeing in position with the stronger unknown line in the cooler peculiar stars. This line was identified by Baxandall with an ionized manganese line which had been observed by him in the laboratory. The line was not observed by Exner and Haschek. It may be due to *Mn* II in α Andromedae, but there can be no doubt that it is not due to that element in stars where the stronger *Mn* II lines are weak or absent.

The star μ Leporis may be taken as of an intermediate type between the group represented by α Andromedae and θ Aurigae. Judging from the level of excitation shown by the commoner elements, it is slightly but distinctly cooler than α Andromedae. The *Fe* II lines are stronger and such *Ti* II lines as λ 4468 and λ 4571, of which only traces are present in α Andromedae, if indeed they are present at all, are easily visible. The helium lines are weaker and the difference between the intensity of *He* I 4471 and *Mg* II 4481 is greater. The *Mn* II lines are still conspicuous and the line at λ 4200 is of approximately the same intensity as in α Andromedae. The strongest line of *Eu* II at λ 4205 is present as a faint but easily visible line which forms a double with *Mn* II 4206, an angstrom and a half toward the red. *Mn* II 4206 is markedly stronger than *Eu* II 4205. The *Si* II doublet, λ 4128 and λ 4131, is much stronger than in α Andromedae. K is about equal to its intensity in α Andromedae.

θ Aurigae and Sirius.—The lines of *Fe* II, *Ti* II, and *Mg* II are weaker in θ Aurigae than in Sirius. It is probable that the temperature of the former is slightly higher than that of the latter. There is an extremely faint blend near the position of *He* I 4471. If helium is

present at all it is at the limit of visibility. The strongest lines of *Fe* I are present, but are rather weak. K is very faint, actually weaker than in β Orionis (gB8) and Maia (dB8). It is fainter than in either α Andromedae or μ Leporis. The strong *Mn* II line at λ 4137 is present but quite faint. The other strong line at λ 4206 is blended with *Eu* II 4205, with which it makes a broad, diffuse line. The unknown line at λ 4200 is considerably stronger than in α Andromedae. The silicon doublet is very strong—much stronger than in Sirius. *Sr* II 4215 is a line of moderate intensity, about the same as in Sirius. The lines of *Cr* II, which up to this type have been rather faint, are well marked and numerous. These lines vary in intensity, but the period and type of variation are not as yet known. The lines in the spectrum of θ Aurigae are slightly broadened by rotation.

The spectrum of the star α^2 Canum Venaticorum was found by Belopolsky more than twenty years ago to contain lines which varied in intensity. A great deal of work has been done on it since that time but the explanation of its spectrum has yet to be found. Since it appears that most of the lines in its spectrum vary in intensity, any description of the spectral characteristics must be referred to some phase. When the europium lines are at maximum intensity the spectrum is packed with fine lines, most of which are of unknown origin. A number of the lines were identified with certain of the rare earths by Kiess. At the maximum phase of *Eu* II, the *Fe* II and *Ti* II lines are stronger than in θ Aurigae, although they are not as strong as in Sirius. The strong *Fe* I lines have about the same intensity as in θ Aurigae. K is stronger than in θ Aurigae, but is still weak for class A0. It is of about the same intensity as *Mg* II 4481. The *Si* II doublet has about the same intensity as in Sirius. If ionized manganese is present, the lines are very weak. *Eu* II, however, is very strong; *Eu* II 4205 is the strongest line in the spectrum in the range $H\beta$–$H\epsilon$ except for the Balmer series of hydrogen. λ 4200 seems to vary oppositely in phase to λ 4205. At minimum of λ 4205, the unidentified line becomes strong. At this time the europium lines become very faint or disappear, the *Si* II doublet grows stronger, and the spectrum becomes the *alter ego* of θ Aurigae. If, therefore, we find that the apparent sequence of peculiar stars is physically a separate group from the normal A-spectra, the spectral changes in α^2 Canum Venati-

corum may become analogous to the changes observed in Cepheid variables among the normal stars. Although most of the peculiar A-type spectrum variables have not as yet been studied sufficiently, it seems possible that they may form a group which corresponds to ordinary Cepheids, except that the variations in light, if present, are very small.

73 Draconis and ϵ Serpentis.—The lines of *Fe* II and *Ti* II are stronger in ϵ Serpentis than in 73 Draconis. K is slightly stronger in the latter star than in α^2 Canum Venaticorum but is much weaker even than in a normal B9 or A0 star. There are many *Fe* I lines in 73 Draconis; these have, however, peculiar relative intensities. *Sr* II 4215 is stronger in 73 Draconis than in ϵ Serpentis. *Cr* II is at its maximum in 73 Draconis. A great many lines of *Cr* I are also present. There is a strong unidentified line at λ 4422 which is present in all the peculiar stars of this type. λ 4200 has almost dropped out of sight. The *Si* II lines are also quite weak; the violet component is blended and the component toward the red is much weaker than in ϵ Serpentis. *Eu* II 4205 is variable in intensity. At minimum it forms part of a diffuse faint blend as in α^2 Canum Venaticorum and θ Aurigae. 73 Draconis, in spite of its many peculiar features, is definitely lower in effective excitation than α^2 Canum Venaticorum.

γ Equulei and μ Ceti.—*Fe* II and *Ti* II are weaker in γ Equulei than in μ Ceti; *Fe* I is of about equal intensity in the two stars. *Sr* II 4077 and 4215 are very strong in γ Equulei and are indeed the strongest lines in the spectrum with the exception of the hydrogen series in the range $H\beta$–$H\epsilon$. No plates of γ Equulei which show K are available. *Cr* II 4558, which is variable in intensity, is at maximum a fairly strong line but is very much weaker than in 73 Draconis. *Eu* II 4129 and 4205 are well marked and are of about the same strength as in 73 Draconis. There is a strong unidentified line at λ 4290.

If the peculiar stars are placed in the sequence according to the manganese, λ 4200, europium, chromium, and strontium lines in their spectra it is found that the stars having abnormally strong silicon lines occur in a narrow range which originates between α^2 Canum Venaticorum (*Eu* II max.) and θ Aurigae and ends between μ Leporis and α Andromedae. It is striking that all the silicon stars would be classed as peculiar on other grounds and could be fitted into

their proper place in the scale without any reference to the intensities of the silicon lines themselves.

Table III classifies those peculiar A-type stars for which spectra are available.

TABLE III

CLASSIFICATION OF PECULIAR STARS

α ANDROMEDAE	
β Tauri	$+33°3154$ Lyrae
γ Canis Majoris	
μ LEPORIS	
53 Tauri	ϕ Herculis
14 Hydrae	υ Herculis
κ Cancri	π Bootis (br.)
θ AURIGAE	
21 Persei	$+33°1008$ Aurigae
τ^9 Eridani	α^2 Canum Venaticorum (min. *Eu* II)
41 Tauri	ι Librae
56 Tauri	4 Cygni
11 Orionis	108 Aquarii
$+73°274$ Camelopardalis	49 Cancri
43 Cassiopeiae	
α^2 CANUM VENATICORUM	
$-18°3789$ Virginis (max. *Eu* II)	45 Herculis
73 DRACONIS	
γ Arietis (S)	84 Ursae Majoris
α Piscium	μ Librae
ι Cassiopeiae	ω Herculis
17 Comae	52 Herculis
21 Comae	10 Aquilae
ϵ Ursae Majoris	κ Piscium
78 Virginis	$-18°3789$ Virginis (max. *Cr* II)
γ EQUULEI	
ω Ophiuchi	β Coronae Borealis

The absolute magnitudes of the peculiar stars.—The spectral evidence is contradictory. The weakening of *Fe* II and *Ti* II and the strengthening of *Sr* II indicate low luminosity, while the enhancement of *Cr* II and *Si* II points to high luminosity. A general idea of the luminosity can be gotten by comparing the widths of the *H* lines. The lines are considerably broadened in all the peculiar standard

stars and are of about the same width as in the normal dwarfs. The absolute magnitudes of the stars as determined from trigonometric parallaxes are given in Table IV.

TABLE IV

	M
α Andromedae	−0.5
μ Leporis	+1.3
α^2 Canum Venaticorum	−0.7
θ Aurigae	−0.8
γ Equulei	+1.3

There is no trigonometric parallax available for 73 Draconis. The mean of the absolute magnitudes of the five stars is $+0^{M}.1$. This value is average for the spectral types considered and shows that the stars do not have either extremely high or extremely low luminosities. This seems to be one more piece of evidence that the peculiar spectra belong to a separate branch of the spectral sequence, as do some of the banded spectra found in the late-type stars.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
January 13, 1933

THE PROBLEM OF TWO BODIES WITH VARIABLE MASSES

By WILLIAM MARKOWITZ

ABSTRACT

This paper discusses the changes in the semi-major axis, a, and the period, P, of a binary picking up matter within a nebula. It is found that a varies inversely as the cube of the sum of the masses M, and, approximately, inversely as the square of the mass ratio μ. P varies inversely as M^5, and inversely as μ^3.

It was found that Jupiter and the sun could not develop into a spectroscopic binary of large mass with mass ratio near unity. The two would fall into each other before this stage could be reached.

What happens when mass is radiated depends upon the assumption made with regard to the process of radiation. For the case in which it is assumed that momentum is conserved when mass is radiated, it is found that if the mass ratio is constant, then a varies inversely as M^3 and P varies inversely as M^5; if it is not constant, then the problem is indeterminate.

I. INCREASING MASS

In 1918 Professor W. D. MacMillan suggested that spectroscopic binaries might have been formed from planetary systems by the picking up of interstellar matter.[1] He found, after assuming that its mass ratio is constant, that the period of a binary varies inversely as the fifth power of the sum of the masses, and that the major axis varies inversely as the third power.[2] We will here derive these same results from a somewhat different point of view, and, in addition, discuss the effect of a change in the mass ratio. The latter is important when discussing the growth of a planetary system. Thus we can hardly consider that Jupiter and the sun will form a binary star until their mass ratio changes from the present value of 0.001 to about unity.

Let a binary star pick up matter during its passage through a nebulous medium. Though it is not essential, we shall assume that all the particles in the nebula are mutually at rest. Let ξ, η, and ζ be a set of rectangular axes fixed with respect to the nebula. Since all the particles in the medium have zero momentum with respect to these axes, it follows that when one of the components of the binary picks up matter its momentum is unaltered. Hence, the changes in the components of momentum of the bodies m_1 and m_2 are due only

[1] *Astrophysical Journal*, **48**, 35, 1918.

[2] *American Mathematical Monthly*, **26**, 326, 1919.

to their mutual gravitative forces. We have, then, for the equations of motion of m_1 and m_2,

$$\left.\begin{aligned} (m_1\xi_1')' &= -\frac{k^2 m_1 m_2(\xi_1-\xi_2)}{r^3}\ ,\\ (m_2\xi_2')' &= -\frac{k^2 m_2 m_1(\xi_2-\xi_1)}{r^3}\ ,\end{aligned}\right\} \tag{1}$$

and four similar equations for the co-ordinates η_1, η_2, ζ_1, ζ_2.

Adding equations (1) gives the momentum integrals

$$\left.\begin{aligned} m_1\xi_1'+m_2\xi_2' = \alpha\ , \qquad m_1\eta_1'+m_2\eta_2' = \beta\ ,\\ m_1\zeta_1'+m_2\zeta_2' = \gamma\ ,\end{aligned}\right\} \tag{2}$$

where α, β, and γ are the constants of integration. Let the relative co-ordinates be introduced by the relations

$$x=\xi_1-\xi_2\ , \quad y=\eta_1-\eta_2\ , \quad z=\zeta_1-\zeta_2\ . \tag{3}$$

By means of (2) and (3) it is possible to express the absolute velocities ξ', η', ζ' in terms of α, β, γ, and the relative velocities x', y', z' as follows:

$$\left.\begin{aligned} M\xi_1' &= \alpha+m_2 x'\ ,\\ M\xi_2' &= \alpha-m_1 x'\ ,\\ M &= m_1+m_2\ .\end{aligned}\right\} \tag{4}$$

After differentiating (1) explicitly it is found, with the aid of (4), that the equations of relative motion are

$$x''=-\frac{k^2 M x}{r^3}-\frac{\alpha}{M}\left(\frac{m_1'}{m_1}-\frac{m_2'}{m_2}\right)-\frac{x'}{M}\left(\frac{m_2 m_1'}{m_1}+\frac{m_1 m_2'}{m_2}\right)\ , \tag{5}$$

and similar equations in y'' and z''. Letting μ be equal to the mass ratio m_2/m_1, it is readily verified that (5) can be written as follows:

$$\left.\begin{aligned} x'' &= -\frac{k^2 M x}{r^3}+\frac{\alpha}{M}\,(\log\mu)'+x'\left[\log\frac{(1+\mu)^2}{\mu M}\right]'\ ,\\ y'' &= -\frac{k^2 M y}{r^3}+\frac{\beta}{M}\,(\log\mu)'+y'\left[\log\frac{(1+\mu)^2}{\mu M}\right]'\ ,\end{aligned}\right\} \tag{6}$$

and a similar equation in z''.

Multiplying the first of (6) by $-y$, the second by x, and adding, we obtain:

$$(xy'-yx')'-(xy'-yx')\left[\log \frac{(1+\mu)^2}{\mu M}\right]'=(\log \mu)'\left(\frac{\beta x-\alpha y}{M}\right). \quad (7)$$

The solutions of this first-order linear differential equation and the two others similarly obtained are

$$\left.\begin{aligned} xy'-yx' &= \frac{(1+\mu)^2}{\mu M}\,(C_1+G_1)\,, \\ yz'-zy' &= \frac{(1+\mu)^2}{\mu M}\,(C_2+G_2)\,, \\ zx'-xz' &= \frac{(1+\mu)^2}{\mu M}\,(C_3+G_3)\,, \end{aligned}\right\} \quad (8)$$

where C_1, C_2, C_3 are the constants of integration,

$$G_1=\int_{t_0}^{t}(\beta x-\alpha y)\,\frac{\mu'}{(1+\mu)^2}\,dt\,, \quad (9)$$

and G_2, G_3 are obtained from G_1 by cyclic permutation of α, β, γ and x, y, z.

We shall now define the instantaneous elements of the orbit as those which the binary would have if at that moment it ceased picking up matter.

The left-hand members of (8) are twice the components of the areal velocity, A, and the sum of their squares equals $4A^2$. But from theorems in celestial mechanics we know that in terms of the instantaneous elements

$$4A^2=pk^2M\,, \qquad p=a(1-e^2)\,, \quad (10)$$

where p is the semi-*latus rectum*, a is the major semi-axis, and e is the eccentricity of the orbital ellipse.

Without any loss in generality we may assume that at time t_0 the $\xi-\eta$ plane is parallel to the plane of motion of the bodies. Then at time t_0, z and z' are both zero, and the constants C_2 and C_3 are also zero. Hence, squaring both sides of (8) and adding gives

$$p=\frac{(1+\mu)^4}{k^2M^3\mu^2}\,[(C_1+G_1)^2+G_2^2+G_3^2]\,. \quad (11)$$

As may be seen from (9), the G's vanish if μ is constant. Hence, if the mass ratio is constant, the semi-*latus rectum* varies inversely as the cube of the sum of the masses. If e does not approach unity, and the resisting medium will actually cause it to approach zero, then we see, from (10), that the semi-major axis also varies, essentially, inversely as the cube of the sum of the masses. By Kepler's law, $P^2 = a^3/k^2M$. Eliminating a from this equation by means of the relation $a = c/M^3$, where c is essentially a constant, gives us the information that the period varies inversely as the fifth power of the sum of the masses. These are the results mentioned above as being obtained by MacMillan.

The G's vanish if (1) μ is a constant; (2) α, β, γ are all zero (this implies that the binary is at rest within the nebula); (3) $t = t_0$; (4) the eccentricity equals zero (this is so because the co-ordinates x, y, and z are, by symmetry, negative and positive for equal periods of time).

Let us suppose, for the moment, that the G's always remain so small compared with the constant of integration C_1 that they may be neglected. Assume, further, that the changes in e are small. Then by means of (10), (11), and Kepler's law it is found that

$$\left.\begin{aligned} \frac{a}{a_0} &= \left(\frac{M_0}{M}\right)^3 \left(\frac{1+\mu}{1+\mu_0}\right)^4 \left(\frac{\mu_0}{\mu}\right)^2 , \\ \frac{P}{P_0} &= \left(\frac{M}{M_0}\right)^5 \left(\frac{1+\mu}{1+\mu_0}\right)^6 \left(\frac{\mu_0}{\mu}\right)^3 . \end{aligned}\right\} \tag{12}$$

A zero indicates the value of the variable at time t_0. Equations (12) indicate that a varies, approximately, inversely as the square of the mass ratio, and P inversely as the cube.

The sun and Jupiter might develop into a binary in several ways. We shall discuss here three cases. In Case I we assume that the sun neither radiates nor picks up any matter, while Jupiter picks up matter only. Equations (12) then apply. They have been used to calculate the values of a and P as μ, whose present value is 0.001, approaches unity. The results are entered in the accompanying table, the distances being given in miles.

μ	0.001	0.01	0.033	0.1	1.0
a	5×10^8	5×10^6	5×10^5	5×10^4	1000
P	4340^d	4^d	3^h	6^m	1^s

Since the radius of the sun is 5×10^5 miles, we see that Jupiter will fall into the sun when its mass becomes about one-thirtieth that of the sun.

In arriving at these results it was assumed that the G's were always negligible compared with C_1. We shall now justify this step. Let the x and y-axes be so orientated that the perihelion point is on the negative x-axis. Then that part of the integral G which contains y as a factor can be neglected because, by symmetry, y will be positive and negative for equal intervals of time. The average value of x, with respect to t, is $(3/2)ea$. We can express a in terms of μ, approximately, by means of (12). In this way we find that

$$G_1 = 1.5\beta e \frac{\mu_0^2}{(1+\mu_0)^4}\int_{t_0}^{t}\left(\frac{M_0}{M}\right)^3 \frac{(1+\mu)^2}{\mu^2}\mu'\, dt\,.$$

If we assume that M is constant, it is then possible to integrate the foregoing, and, neglecting powers of μ_0 higher than the first, it is found that $G_0 = 1.5\, a_0 e\beta\mu_0$.

By means of (11), (10), and Kepler's law it is found that C_1 can be written as follows: $C_1 = (2\pi a_0/P_0)a_0\mu_0(1-e^2)^{\frac{1}{2}}M_0/(1+\mu_0)^2$. The first factor on the right, which we shall designate v_0, is approximately the orbital velocity of Jupiter at t_0. Neglecting e^2, and μ_0 when it occurs in the denominator, we see that $C_1 = v_0 M_0\mu_0 a_0$.

If the sun is taken for the unit of mass, and the kilometer per second for the unit of velocity, then $v_0 = 10$. If the solar system passes through a nebula with a velocity of about 20 km/sec., then β will also have a value of about 10. The eccentricity of Jupiter's orbit is 0.05. Hence, on comparing the numerical values of C_1 and G_1, we find that the former will always be several times larger than the latter. The integrals G_2 and G_3 are of the same order of magnitude as G_1 and can also be neglected. Our previous results are not changed materially even if we apply equations (12) to a planetary system of large eccentricity, for even if the G's attain the same order of magnitude as C_1, the values of a and P are very nearly those given by (12), as may be seen by examining (11).

In case II we assume that the sun is picking up matter at the same rate at which it is radiating it away, so that its mass, m_1, is constant. Assume that E, the rate of gain, and also the rate of loss of mass is constant. We assume, further, that Jupiter picks up matter only, and at a rate, in comparison with the sun, proportionate to its mass. Hence, m_2 being the mass of Jupiter,

$$\frac{dm_2}{dt}\frac{1}{m_2}=\frac{E}{m_1}\,. \tag{13}$$

Letting $\mu=m_2/m_1$ as before, we find, after integrating (13), that

$$\mu=\mu_0 e^{\frac{E}{m_1}t}\,. \tag{14}$$

If (13) is satisfied, then there is no change in the mass ratio due to the picking-up of matter. Hence, when mass is picked up, $a=k_1/M^3$, where k_1 is a constant. Differentiating logarithmically, we find that the rate of change of a due to the accretion of matter is

$$\frac{da}{dt}\cdot\frac{1}{a}=-\frac{3}{M}\left(E+\frac{dm_2}{dt}\right)\,. \tag{15}$$

The quantity within the parentheses is the rate at which mass is being picked up.

The change in a due to the radiation of mass cannot, at present, be definitely determined, but under certain reasonable assumptions it is found that when mass is radiated $a=k_2/M$, k_2 being a constant. Changes in the mass ratio, under these assumptions, do not affect a. If we accept this equation, then it is found, upon differentiation, that the rate of change of a to the radiation of mass is

$$\frac{da}{dt}\cdot\frac{1}{a}=-\frac{1}{M}\,(-E)\,. \tag{16}$$

The total rate of change of a is obtained by adding (15) and (16). Adding, making use of (13), and the fact that $M=m_1(1+\mu)$ show that the total rate of change is

$$\frac{da}{dt}\frac{1}{a}=-\frac{2E}{m_1}-\frac{E}{m_1}\frac{\mu}{(1+\mu)}\,. \tag{17}$$

By means of (14), μ can be expressed in terms of t. If this is done, then (17) can be integrated, and it is found that

$$\frac{a}{a_0}=\left(\frac{1+\mu_0}{1+\mu}\right)\left(\frac{\mu_0}{\mu}\right)^2 . \tag{18}$$

Here again a varies, approximately, inversely as the square of the mass ratio. Hence, if a table were constructed giving a and P in terms of μ, we would get the same values as in our previous table. Thus, again Jupiter would fall into the sun when its mass became about one-thirtieth that of the sun's.

In case III we assume that the sun decreases in mass until it becomes equal to that of Jupiter. We now have a binary with mass-ratio unity and total mass 0.002. The value of a for this system is given by the formula $a=k_2/M$.

Let the system now increase in mass until each component has a mass of 1.0. The value of a is given by the formula $a=k_1/M^3$. We now have a binary of total mass 2.0 and mass ratio 1.0. It is found, upon using the given formulae, that for this system a equals 250 miles.

Upon further computation it is found that Jupiter and the sun would have fallen into each other when the mass of each reached 0.1.

SUMMARY

Three ways in which Jupiter and the sun could grow into a binary star were investigated. If this development is to take place under the most favorable conditions, it must do so according to one of these methods or in some combination of them.

It was found that they will not form a binary of the type usually observed, that is, with mass ratio near unity and with total mass equal to or greater than the sun's. It was found that Jupiter and the sun would fall into each other when the mass ratio reached one-thirtieth, in two cases, and when the total mass became 0.2 in the other.

On the basis of these results, it is conjectured that (a) the initial value of the mass ratio must be at least one-fiftieth in order that a typical spectroscopic binary of very short period may develop from

a planetary system and (*b*) massive binaries and binaries with periods larger than ten days have probably not had their origin in planetary systems.

II. DECREASE OF MASS

The problem we discussed in the last section, that of two bodies with masses increasing by the accretion of matter, is one to which the ordinary principles of Newtonian mechanics are applicable. This is not the case, however, with the present problem, that of a decrease of mass by annihilation of matter. For example, the velocity of a body may remain constant when mass is radiated, or its momentum may remain constant. These are perhaps the two most natural possibilities to consider, but they are not necessarily the only ones. As equivalent conditions, we may state the law of motion for a body losing mass.

The problem of two bodies with diminishing mass when the assumed law of motion is Force = Mass × Acceleration has been investigated by several writers, and it was found that a varies inversely as M and that P varies inversely as M^2. It is convenient to discuss the case here in which the law of motion is Force = Rate of Change of Momentum, because the equations of motion are equations (1) of section i. It should be noticed, however, that our fixed set of axes was referred in the previous case to a nebula. In the present instance we have no particular set of axes to single out, and the constants α, β, and γ are therefore indeterminate.

The equations of relative motion (6) contain the terms α, β, and γ, except for the particular case in which the mass ratio is a constant. If μ is a constant, then it is possible to integrate the equations as before. Hence, we have the following results for the problem of two bodies with diminishing masses when the law of motion assumed is Force = Rate of Change of Momentum.

The problem is indeterminate unless the mass ratio is constant. If the latter condition is satisfied, then a varies inversely as M^3, and P varies inversely as M^5.

UNIVERSITY OF CHICAGO
January 14, 1933

NOTES

SCANDIUM OXIDE BANDS IN STELLAR SPECTRA

ABSTRACT

Scandium oxide bands at λ 6036 and λ 6072 have been found in the spectra of seven stars, beginning with class K5. The bands increase in intensity from K5 to M3.

F. E. Baxandall[1] identified certain bands in the spectrum of *o* Ceti as due to the molecule *ScO*. In the reproduction of the spectrum of this star given by him, a good correspondence between the laboratory and stellar wave-lengths can be seen. However, in most cases this correspondence is questionable as the bands, if present at all, must be blended with strong atomic lines of almost identical wave-lengths as the heads of the bands.

The best laboratory data on the *ScO* bands have been published by W. F. Meggers and J. A. Wheeler.[2] In the region investigated by Baxandall, four systems of *ScO* are present, denoted by the authors as II, III, IV, and V. In reality there are only two systems, but each of them gives double heads. According to Meggers and Wheeler, the electronic transitions are as tabulated.

System	ν of the (0, 0) Head	Branch	
II	16,615 cm^{-1}	R	$^2\Pi_{3/2}-{}^2\Sigma_{1/2}$
III	16,562	Q	
IV	16,485	R	$^2\Pi_{1/2}-{}^2\Sigma_{1/2}$
V	16,445	Q	

The laboratory wave-lengths and intensities of the strongest heads and the same data for the neighboring solar lines are given in Table I. All other heads of the extremely rich spectrum of *ScO* are very weak.

It is seen that the only reliable identification may be made for the bands of *ScO* at λ 6036.17 (System III, band 0, 0) and λ 6072.65 (System III, band 1, 1). In all other cases the bands will be masked by the strong spot lines, all of which are very strong in the late-type spectra. Moreover, unless λ 6036.17 is very strong, we should not

[1] *Publications of the Astronomical Society of the Pacific*, **41**, 168, 1929.

[2] *Bureau of Standards Journal of Research*, **6**, 239, 1931.

expect the appearance of the other *ScO* bands in stellar spectra. On Baxandall's reproductions the ratio of intensities of the 0, 0 band to the 1, 1 band is as 3:1, whereas other supposed bands are much stronger than the 1, 1 band. This in itself throws doubt on his identification. The band at λ 6036.17, and possibly λ 6072.65, should,

TABLE I

λ I.A.	i	$\lambda_{\odot}$	i Disk	i Spot	Element
6036.17......	200	....	....	....	....
6064.31......	80	{6064.65 6065.50	−1 7	4 8	*Ti* *Fe*
6072.65......	100	....	....	....	....
6079.30......	100	6078.50	5	5	*Fe*
6101.87......	30	{6102.19 6102.73	6 9	5 25	*Fe* *Ca*
6109.93......	40	{6108.13 6111.67	6 0	8 10	*Ni* *V*
6115.97.....	40	6116.19	4	4	*Ni*
6140.32......	10	{6137.71 6141.73	7 7	9 12	*Fe* Ba^+
6148.70......	20	6150.11	0	12	*V*
6153.93......	20	6154.24	2	7	*Na*

however, be easily identified in the late-type spectra if *ScO* is present at all.

A search for these bands was made on spectrograms obtained with the 69-inch Perkins reflector and an auto-collimating grating spectrograph constructed by the Yerkes Observatory. The average dispersion of this instrument is 26.7 A per millimeter. The bands in question were measured in respect to two pairs of lines on both sides of the bands. These lines were *V* 6039.75 (*i* 2) and *Ti* 6031.72 (*i* 3) for the band 6036.17, and *Ti* 6085.26 (*i* 2) and *Fe* 6065.50 (*i* 5) for the band λ 6072.65. The relative intensities of these lines (estimated on an arbitrary scale) remain approximately constant for classes K5–M3.

The results of measurement are given in Table II.

The weight of individual determinations was assumed to be equal to the number of plates on which the bands in question were measured. The probable errors are not large for faint and diffuse bands. In most cases it was possible to see that the bands were degraded to the red, so that the micrometer wire was set on their violet edge.

It appears that the *ScO* bands are present in the spectra of the late-type stars. No trace of them could be found in the spectrum of Arcturus. They begin to appear in class K5 and increase in intensity very considerably from K5 to M3; in other words, they are very

TABLE II

Star	Class	λ	Weight	*i*	λ	Weight	*i*
α Tauri	K5	6036.36	1	0.3	6072.65	1	0.1
40 Lyncis	Mo	6035.88	2	0.3	......	......	......
δ Ophiuchi	Mo	6035.86	1	1.5	......	......	......
β Andromedae	Mo	6036.22	2	0.5	......	......	......
α Orionis	M2	6036.20	4	1 and 0.5	6072.72	1	0.5
α Ceti	M2	6036.44	1	2	6072.15	1	1
η Geminorum	M3	6036.29	2	2	6072.96	2	1
Weighted mean	......	6036.17±	0.05	......	6072.69	±0.14	......
Laboratory	......	6036.17		......	6072.65		......

sensitive to the comparatively small change in temperature between classes K5 and M3. The heat of dissociation of the *ScO* molecule must therefore be rather low, much lower than that of *TiO*. Unfortunately, I could find no data on this important point.

The appearance of the *ScO* bands only in class K5 explains their absence from the solar spectrum. Richardson[3] could not identify them in the sun-spot spectrum, although a special search was made for them.

The spectral classification for the M-type stars is that of Mount Wilson.[4] 40 Lyncis is usually described as a K5 star; the strength of the *ScO* bands in its spectrum is about normal for this type.

In α Orionis the *ScO* bands on plates obtained on November 27, 1932, and on January 14, 1933, are much stronger than on subse-

[3] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 422, 1931.

[4] *Ibid.* No. 319, 1926.

quent plates obtained on January 30 (two plates) and February 11, 1933. The effect seems to be real. No such effect was observed in the spectrum of the variable η Geminorum.

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A NOTE ON THE INTENSITIES OF *He* I IN P CYGNI

ABSTRACT

The total areas for absorption and emission of λ 4388 and λ 4472 of the *He* I spectra of P Cygni are given from measurements of eight plates. The ratio of the averages of λ 4472 abs/λ 4388 abs = 1.1 while λ 4472 em/λ 4388 em = 9.8.

Microphotometer tracings of eight standardized plates of 34 P Cygni (H.D. 193237, B1p) taken with single-prism dispersion on Eastman-40 emulsion were reduced in the usual manner. The results for λ 4388 (2^1P-5^1D) and λ 4472 (2^3P-4^3D) are given in Table I.

TABLE I

DATE; G.C.T.	AREAS				C_1/C_3
	Absorption		Emission		
	4388	4472	4388	4472	
1928 Sept. 3.18	0.48	0.77	0.13	1.23	0.86
Sept. 21.08	.67	0.48	.08	1.66	.86
1929 March 18.42	.83	0.95	.13	1.32	.86
June 4.30	.84	0.70	.10	1.30	.85
July 2.26	.74	1.05	.34	0.94	.61
July 29.26	.53	0.55	.04	2.25	.87
1930 March 28.45	.28	0.46	.32	1.93	.78
July 8.20	0.58	0.55	0.11	1.81	0.82
Mean	0.62	0.69	0.16	1.56	0.81

These areas are in units of 100 per cent absorption or emission per angstrom with respect to the intensity of the continuous background of the star's spectrum at each line respectively. The linear dispersion of the spectrograms was 30 A per millimeter at λ 4500.

The column C_1/C_3 gives the ratios of intensity of the continuous background at λ 4388 to that at λ 4472. The problem of estimating the position of the continuous background in this star is somewhat more arbitrary than usual, because of the red displacement of the emission lines with respect to their absorption lines.

Disregarding possible changes from plate to plate, the ratio of the average area of λ 4472 to that of λ 4388 is 1.1 in absorption and 9.8 in emission.

Since it is considered by C. S. Beals[1] and others that the emission and the violet absorption lines in P Cygni are produced in the same expanding shells of gas, this difference between the two helium lines in the ratios of the average areas for emission and absorption is of interest.

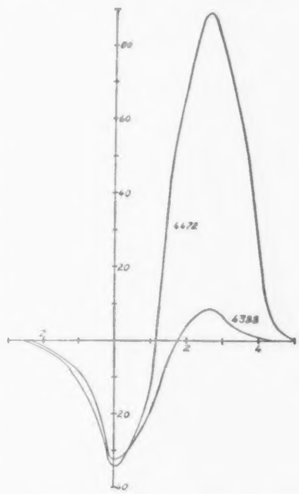

FIG. 1.—Contours of helium lines in P Cygni.

The ratio of the intensities in emission of the triplet to the singlet line for this star is appreciably greater than that observed in absorption and is also greater than the ratios obtained from the usual visual estimates of intensities in the ordinary discharge tubes. Though recombination enhances the ratio of triplet to singlet lines in helium,[2] the amount ascribable to this cause in P Cygni is problematical at present.

In view of the results obtained by O. Struve concerning the intensities of *He* I absorption in B-type stars,[3] the average of the ratios of λ 4388 to λ 4472 for absorption, derived from each plate of this star, falls at its spectral type B1.[4] On some plates, however, the total absorption of λ 4388 appears to be greater than that of λ 4472.

I wish to express my appreciation to the Yerkes Observatory and to Dr. Struve for permission to make and reduce these microphotometer tracings.

HAROLD F. SCHWEDE

CHICAGO, ILL.
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[1] *Monthly Notices of the Royal Astronomical Society*, **90**, 202, 1929.

[2] F. L. Mohler, *Review of Modern Physics*, **1**, 216, 1929.

[3] *Astrophysical Journal*, **74**, 225, 1931.

[4] In this respect it is of interest to note a determination made by C. S. Beals of a minimum of 25,000° for the temperature of P Cygni from the use of the emission lines of the first two members (λ 5876 and λ 4471) of the diffuse triplet series of the *He* I spectra (see Beals, *op. cit.*, p. 677, 1932).

REVIEWS

Atomic Energy States as Derived from the Analysis of Optical Spectra. By ROBERT F. BACHER and SAMUEL GOUDSMIT. New York: McGraw-Hill Book Co., 1932. Pp. xiii+562. $6.00.

Ten years ago Catalàn's discovery of multiplets in the spectrum of manganese opened the way to the long-sought understanding of complex atomic spectra. The importance of the subject, the wealth of precise data available, and a certain fascination in the work (curiously akin to the attraction of crossword puzzles) put a large number of investigators into the field. Within five years the theory of atomic spectra had developed so far as to account for the characteristics of all the energy-levels that were then known to exist in atoms, and the same success has been maintained since. The present volume undertakes only the task of listing the energy states which had been identified in all atoms up to the spring of 1931—and to do this fills more than five hundred pages!

This limitation of scope makes it of value to the theoretical student of atomic structure rather than to the astrophysicist. Could it have been expanded to include the lines produced by transitions between the various energy states, it would have had a wider usefulness; but it would have grown to many times its bulk, and have been beyond the power of any one or two authors to compile in a reasonable time.

An introduction of twenty-one pages gives an admirable outline of the theory of spectral structure—too condensed for the beginner, but very useful to refresh the memory of the experienced student. The tables which form the bulk of the volume give the energy-levels (in frequency units) with the spectroscopic notation, as now generally adopted, whenever this can be assigned with reasonable certainty. Odd levels are distinguished by numbers in italics—a great convenience. The levels are measured upward from the lowest state in complex spectra, and downward from the series limits when these are accurately known, and Paschen's notation for the neon spectrum (which is in many respects convenient) is retained. References are given to the most recent and complete work on each spectrum, which should help the student interested in the lines rather than the terms. Very good judgment has been shown in selecting these authorities, and in the occasional notes upon doubtful cases. In many cases unpublished material communicated by the investigators is included.

All told, data are given for 68 elements. The arc spectra have been more or less completely analyzed in 66 cases, the first spark spectra in 57, and one or more of the higher spectra in 45.

For the latter, however, the analysis is usually very incomplete—only a few prominent lines in the far ultra-violet being classified. A survey of the data (by the reviewer) indicates that a substantially complete interpretation has been made for the arc spectra of some 42 elements, the first spark spectra of 28, and higher spectra of 10 (including fortuntately those of greatest astrophysical importance). For about 20 more arc spectra, and as many of singly ionized atoms, the analysis has progressed far enough to include most if not all of the lines of astrophysical interest.

The compilation appears to be very complete, and a supplementary list of more than a hundred references carries the survey (though not the tabulation) well into the present year.

The work concludes with tables of constants (Birge's values) and a reprint of Paschen's *Rydberg Term Tables*. The latter are given separately for the first four stages of ionization (two pages for each), but the tabular interval (0.05) is so wide that interpolation is hardly practicable. It would have been better to give a single table, for neutral atoms, with a smaller tabular interval.

Only one other criticism of this excellent work need be made. In the Introduction the word "multiplet" is used consistently to denote a group of closely related energy-levels, such as is called by most spectroscopists a "multiple term" (e.g., ^5H). Catalàn's usage of multiplet to describe the group of spectral lines arising from the combination of two multiple terms has clear priority, and has been very widely adopted by astrophysicists as well as spectroscopists. To use the same word with another meaning leads to needless confusion.

The paper and presswork are so good that the authors may well be envied by those who have been obliged to use less aesthetic methods of publication during the present distress.

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PRINCETON UNIVERSITY OBSERVATORY
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Optik: Ein Lehrbuch der elektromagnetischen Lichttheorie. By MAX BORN. Berlin: Julius Springer, 1933. Pp. vii+591; Figs. 252. RM. 38.

The day is past when the subjects of physical science can be distinguished by sharp definitions. This must be counted an advance in the

state of the science, but to an author it may be a source of perplexity. The boundary of the subject of optics as treated by Professor Born would appear strange, indeed, to Newton, who did not recognize the independent status of spectroscopy; to Navier and Cauchy, who sought its foundations in the theory of elasticity; and even to the phyicist of the beginning of the present century the omission of the optics of moving mediums would seem unpardonable. Today the omission of all these topics from a treatise on optics may be condoned but scarcely applauded. It is true that spectroscopy may safely be left to its specialists, but few physicists would agree to cede the optics of moving mediums to the modern relativist, whose interests all too often center on speculative cosmology. There are also many who will disagree with the dictum, expressed in a scholarly but tantalizingly brief historical survey, that discussions of the difficulties of the ether theories are now quite superfluous. To act on this principle in teaching the present generation of theoretical physicists would be to make shallow sophisticates of them, and shortly to replace the present healthy skepticism with a scholasticism as hopeless as any that science has yet known.

But if the rather arbitrary choice of topics makes the *Lehrbuch* unsuitable as a textbook for a well-rounded course on optics, the advanced reader will find no difficulty in pardoning it. For the author has been vitally interested in all the topics treated, and has given a critical and authoritative exposition of physical theory whose scientific honesty cannot be praised too highly. Nowhere is the desire for plausibility and short-cuts allowed to interfere with a clear and leisurely statement of the difficulties involved. This is as true in the first half of the book, which treats of the classical topics of plane waves, geometric optics, interference and diffraction, as in the second half, where the interest centers on the clear delineation of the boundary between those regions in which quantum principles are essential and those in which they are not.

The treatment of plane waves, as its author points out, follows closely along accepted lines. So does the treatment of interference, except that it is somewhat more sharply distinguished from diffraction than is usual in texts which tend to follow the historic order of experimental rather than conceptual simplicity, and present diffraction before taking up interference in detail. The chapter on "Diffraction" contains much new material. It begins with a clear account of the researches of Kirchhoff on the relation of Huyghen's principle to the wave equation of physical optics. After taking up the usual topics, Debye's rigorous theory of diffraction in the neighborhood of a focal point is presented and extended to a general ac-

count of the influence of diffraction on the errors of optical instruments; this last is original and hitherto unpublished work. The chapter closes with Sommerfeld's rigorous treatment of diffraction by a semi-infinite screen.

The chapter on "Geometric Optics" is systematic and theoretical in character. It is largely devoted to Hamilton's characteristic function and the eikonals (a more specific reference to the relation of these functions to the canonic transformations of mechanics would have been desirable); these functions are used to give an exhaustive treatment of the errors of optical images up to and including the third order.

The optics of crystals and metals receive a standard treatment in the fifth and sixth chapters, which is then supplemented by the Ewald-Born theory, etc., in the next chapter on "Molecular Optics." It would be dangerous to attempt a valid summary of this and the last chapter, on "Emission, Absorption and Dispersion." Together they make up almost half the book, and contain much material that has heretofore been included in texts on electron theory, and much that has not previously been available outside the journals or separate monographs. The sections on the width of emission lines and on absorption will be of particular interest to astrophysicists. The laws of quantum mechanics are mentioned, but, as stated above, the aim is merely to carry the treatment to the point where these become inescapable. The quantum theorist may occasionally be surprised at the distance of this point, e.g., the sections on the Raman effect. A very valuable section on the vibrations of polyatomic molecules is included.

CARL ECKART

Table of Arc Spectrum Lines Arranged in Order of Wave-Length 2785 A.U.—3505 A.U. By WELTON J. CROOK. Palo Alto: Stanford University Press, 1933. Pp. 30.

The author of this table has collected data from a number of sources for the purpose of including all the known spectrum lines of metallurgical importance between wave-lengths 2785 and 3505 A. This spectral region is chosen because of its general usefulness and particularly for the analysis of impurities in steel and ferrous alloys. The table differs from Kayser's "Tabelle der Hauptlinien" in that it includes as many of the very faint lines of the elements as the author could find in the literature. In addition, a large number of tungsten and vanadium lines are included, the wave-lengths of which were determined by the author. The wave-lengths are given to one-tenth Angstrom unit. The general value of the

table would have been much enhanced had they been recorded to three decimal places, especially in the case of the secondary standards.

F. E. Roach

Die Quantenstatistik und ihre Anwendung auf die Elektronentheorie der Metalle. Struktur der Materie in Einzeldarstellungen. Volume 13. By Léon Brillouin. Translated from the French by E. Rabinowitsch. Berlin: J. Springer, 1931. Pp. x+530; Figs. 57. RM. 43.80 net.

The most valuable feature of this book is the detailed account which it gives of the present status of the theory of metallic conduction and related effects. Although this is a subject that has been placed on a sound theoretical basis by the pioneer work of Sommerfeld, which is followed closely in the text, certain aspects of it still present unsolved problems of the greatest interest. These have to do largely with the wave mechanical calculation of the "mean free path" of an electron in a crystal lattice, and 150 pages are devoted to a detailed and critical review of the literature of this controversial subject.

The first six chapters, on the other hand, are more general in character and contain an account of modern statistical theory and related topics, including a sketch of the quantum mechanics extending even to the quantization of the radiation field. Unfortunately a certain diffuseness of style, which is in contrast to the precision and clarity of the more special discussions, makes this portion of the text difficult to assimilate. The important topics treated here include the theory of black body radiation, energy fluctuations, the ideal gas, paramagnetism, and the H-theorem.

The book closes with a chapter on the application of Fermi statistics to the electron distribution in heavy atoms and one on ionization and dissociation. There is no discussion of astrophysical applications.

The German edition differs materially from the French, not only in the revision of several portions of the text, but also in the inclusion of a large amount of new material.

F. C. Hoyt

INDEX TO VOLUME LXXVII

SUBJECTS

INDEX TO VOLUME LXXVII

AUTHORS